A Method for Working With Sign Restrictions in Structural Equation Modelling*

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Abstract

The paper sets out a method for handling sign restrictions in systems of simultaneous equations which are only partially identified. These sign restrictions might apply to either structural equation parameters or functions of them such as impulse responses. Initially a range of values for the unidentified parameters are generated and then the role of sign restrictions is to narrow the range. It is simple to apply and can be handled in packages such as EViews and Stata. Examples are given of how to implement it in a number of cases where there are both parametric and sign restrictions.

1 Introduction

Consider a system of n structural equations in an $n \times 1$ vector of variables y_t with p lags. This will be written as

$$A_0 y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + B_0 \eta_t, \tag{1}$$

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where η_t is a $n \times 1$ vector of shocks taken to be $N(0, I_n)$, while $\varepsilon_t = B_0\eta_t$ will contain the structural equation shocks. Exogenous variables can be subsumed into y_t by appropriate definitions of A_0 and B_0. In the traditional simultaneous equations format estimation of the unknown parameters was done by imposing zero restrictions on the A_j. In the exactly-identified Structural Vector Autoregression (SVAR) case $A_1, .., A_p$ were left unconstrained and restrictions were imposed on A_0 and B_0. Defining $A = A_0$ and $B = B_0$ one gets the form of SVARs pioneered by Amisano and Giannini (1997) and used in programs such as EViews and Stata. We refer to this as the (A, B) technology and will use it extensively in this paper.

In most structural model applications A_0 has been taken to have unity on its diagonal (reflecting a normalization) and B_0 to be a diagonal matrix containing the standard deviations of the shocks. In such a form there are n^2 unknown elements in A_0 and B_0. Accordingly, to estimate these parameters requires some moment conditions. $\frac{n(n+1)}{2}$ of these are available using the fact that the shocks $\varepsilon_t = B_0\eta_t$ are uncorrelated. This leaves $n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$ "free" parameters whose values need to be found by some method. Arranging these parameters in a $\frac{n(n-1)}{2} \times 1$ vector β, this vector is estimated in SVAR work by imposing $\frac{n(n-1)}{2}$ restrictions on A_0, e.g. Sims (1980) made A_0 triangular.

There have of course been other suggestions for setting these $\frac{n(n-1)}{2}$ restrictions. One of these involving A_0 and the A_j $(j > 0)$ is the long-run restriction employed by Blanchard and Quah (1999). However, in many instances one might only have K plausible parametric restrictions, where $K < \frac{n(n-1)}{2}$. In those situations it is only possible to estimate K of the unknown parameters, leaving $\frac{n(n-1)}{2} - K$ that are unidentified. Consequently, β might be divided into a $K \times 1$ vector β_1 of estimable parameters leaving β_2 to capture the remainder, namely those that are unidentified.

In this paper we suggest that values for the parameters β_2 be generated by some procedure, following which β_1 can be estimated. This will be done conditional upon the generated values of β_2 and by using the K restrictions on A_j. Clearly β_1 will not be unique and there will be a range of values for it. To discriminate between these one might use some extra information. A popular example of this - often described as "agnostic" - has been the use of sign information. This might come directly from the β_1 and β_2 parameters themselves but mostly it has been about some functions of them, such as impulse responses. This will narrow the range, although it will rarely make it possible to get a unique set of values for β_1. Nevertheless sign restrictions on impulse responses in SVARs have become a very popular way of doing emprical work. They do not provide a single set of impulse responses but a range of possible outcomes. There are many applications of this methodology, e.g. Canova and De Nicoló (2002), Elekdag and Han (2015), Faust (1998), Jääskelä and Jennings (2011), Uhlig (2005), Mumtaz and Zanetti (2012), and Stângă (2014).

Section 2 sets out our method for finding a range of values for β_1 and β_2 (and hence statistics that might be constructed from them) in the context of a simple demand and supply system that we term the market model. In Section 3 the ideas are applied to larger systems, along with a number of types of parametric restrictions, i.e. $K \neq 0$. Our strategy will be to formulate the system so as to be able to use the (A, B) technology. Some of the parameters in A_j, B_0 can be estimated using parametric restrictions, namely β_1, but there will be others that cannot (β_2), and we therefore propose that they be generated by some mechanism. The use of the (A, B) technology means that our method can mostly be implemented with programs such as EViews and Stata, hence justifying the "simple" descriptor in the title of the paper. Our method will be designated as SRC - sign restrictions with generated coefficients.

Section 4 considers how our method relates to the sign restriction approach used in the SVAR literature. This was begun by Faust (1998), Canova and De Nicoló (2002) and Uhlig (2005), but has recently been given a general treatment in Arias *et al.* (2014). Because the examples of Section 3 mostly involved signs of impulse responses, we can ask what the advantage of our method would be over the method proposed in Arias *et al.* It is argued that our method is relatively easy to implement because of its use of existing software such as EViews and Stata; it can be applied in a wider context than SVARs; and is possibly more transparent, which is useful for teaching and communication. Lastly, Section 5 concludes. In it we note that, although our focus has been upon SVARs, the method we advocate can be applied equally well to any context which involves a simultaneous equation set-up. Since microeconomic studies often have this structure it therefore has potential application in that work as well.

2 The SRC method in a simple demand/supply context

Suppose we had the classic demand and supply model in (2)-(3) where the shocks ε_{jt} are uncorrelated with standard deviations σ_j. This will be termed the market model.

$$q_t = \alpha p_t + \varepsilon_{1t} \quad (2)$$
$$q_t = \delta p_t + \varepsilon_{2t}. \quad (3)$$

In terms of the system of the introduction $A_0 = \begin{bmatrix} 1 & -\alpha \\ 1 & -\delta \end{bmatrix}$, $B_0 = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$ and $A_j = 0$ $(j > 0)$. Since $n = 2$ there are $n^2 = 4$ unknown parameters - α, δ, σ_1 and σ_2. The restriction that the shocks are uncorrelated produces three moment condition $E(\varepsilon_{1t}^2) = \sigma_1^2$, $E(\varepsilon_{2t}^2) = \sigma_2^2$ and $E(\varepsilon_{1t}\varepsilon_{2t}) = 0$. These would enable the estimation of $\frac{n(n+1)}{2} = 3$ parameters. Assuming that two of these are σ_1 and σ_2 means that it is only possible to estimate one of the remaining two coefficients α and δ.

A single parametric restriction $(K = 1)$ such as $\alpha = 0$ would enable the estimation of δ by OLS. Suppose however that this is not a plausible restriction and that there are no others. Then $K = 0$. Some extra information is available however, namely that the signs of α and δ must be opposite, if one of the equations is to be a demand and the other a supply curve. Therefore our method to exploit this sign information would be to proceed as follows.

(i) Generate some value for α (this is the equivalent of β_2 in the introduction) and call it α^*.

(ii) Then, with $\alpha = \alpha^*$, apply MLE to (2) and (3) to estimate δ, σ_1, σ_2 (these parameters are the equivalent of β_1).

There is an alternative approach to getting the ML estimate of δ that is instructive. Because the system is exactly identified, once α is fixed at α^* the MLE of δ^* is identical to the instrumental variable estimator of δ using the instrument $(q_t - \alpha^* p_t)$ for p_t in (3) (see Durbin (1954) and Hausman(1975)).[1]

[1] Note that the IV estimator is found using the moment condition $E(\varepsilon_{1t}\varepsilon_{2t}) = 0$ and so enforces uncorrelated shocks. The MLE uses this restriction when setting up the likelihood.

Clearly, for every α^* value we get a δ^* estimate, so there will be a range of values for the pairs (α^*, δ^*). Some of these pairs can be rejected, namely those that have the same signs. Consequently, this is how the sign restriction information is to be used, i.e. first produce a range of values for the coefficients α, δ that are compatible with uncorrelated shocks, followed by rejection of some of these based on their signs. If one wishes to narrow this range even further then extra information of some sort would be needed, e.g. one might think that the price elasticities for supply and demand should not exceed certain values. There is nothing in what we propose that stops the use of such information, since it occurs *after* estimation is done by MLE. One would simply reject those values that lie outside the postulated range as well as those with incorrect signs. It is convenient however if we just give the generic description to the type of information to be used to narrow the range as "sign information".

Now if there were dynamics in the equations above and nothing is known about the coefficients on the dynamics, i.e. A_j $(j > 0)$ is unrestricted, then one might work with *functions* of A_0 and A_1. A particular function would be impulse responses. For the static system of (2) and (3) the response of q_t to a unit shock in ε_{2t} is $\frac{\alpha}{\alpha-\delta}$, i.e. once α and δ values are found impulse responses can be computed.[2] Sign information might be available on this impulse response. Table 1 gives what would be the most likely responses of prices and quantity to *positive* demand and cost shocks in the market model.[3] Clearly we can use α^*, δ^* to compute a range of impulse responses whose signs can be compared to those in Table 1. This enables us to identify which of ε_{1t} and ε_{2t} is the demand and which is the cost shock. Of course it is possible that there are no shocks with impulse responses of the correct sign, in which case we would reject those values of α^* and β^*, just as was done when the information pertained to the signs of the coefficients.

[2] Of course if there were dynamics in the market model A_1 would be computed and the impulse responses would be formed from it as well as α and δ. But this poses no difficulties for MLE, and is standard in programs such as EViews, provided there are no restrictions on A_1.

[3] One has to allow for the fact that shocks could be negative rather than positive or one might be negative and the other positive. In those situations the sign patterns are clearly different to what is in Table 1. This was discussed in Fry and Pagan (2011) and we just take that as given here, always referring to the signs of impulse responses for positive shocks. In empirical work when deciding on whether a particular set of impulse responses is accepted we take into account the need to examine all combinations of the type of shock.

TABLE 1		
Sign restrictions for the market model		
(positive demand/supply (productivity) shocks)		
Variable\shock	*Demand*	*Supply*
q_t	+	+
p_t	+	-

This leaves the question of how α^{*}· is to be generated? Basically we need a mechanism for ensuring that the space of values for α is sampled as effectively as possible. This is in order to find all the values of δ that are compatible with those values of α, and which also satisfy the constraint of uncorrelated shocks. In the event that $\alpha > 0$ we would generate a θ from a uniform (0,1) (U(0,1)) density and then write $\alpha = \frac{\theta}{(1-abs(\theta))}$. The same formula is used if the sign of α is unknown, but now θ will be drawn from a U(-1,1) density. This latter case arises when the sign restrictions are on impulse responses rather than the structural coefficients. Of course there may be other ways of generating α, such as from some uniform density over (0,G) or $(-G, G)$, but we would need to specify a value for G. It should be clear that, given a set of data, $\hat{\delta}$ is a function of α, and so the density of $\hat{\delta}$ *across models* will be a function of how α is generated. Thus something like the median of $\hat{\delta}$ will be affected. The range of $\hat{\delta}$ however is more dependent on how complete the sampling of the space of α is.

Once a value of θ is generated this will fix α to some value α^{*}. It is clear then why we call our method SRC, as some parameters (α here but β_2 more generally) are generated, the remaining parameters are estimated, and then sign information is used to narrow the range of outcomes for the parameters β_1 and β_2. Standard errors will come from the method used for estimation. These will be conditioned upon a particular draw of α^{*} which, along with $\hat{\delta}(\alpha^{*})$, will be described as producing a model. Although programs which use the (A, B) technology generally use ML for estimation it could be done by any method that will estimate the parameters of a structural system. Consequently, for any given α^{*} there will be a standard error for the remaining coefficients that are estimated (δ in the market model). These standard errors arise due to the fact that data is used to estimate some parameters. There is also a spread in the estimates of δ which stems from the fact that there is not a unique α^{*}, and so there are many models that are feasible (different α^*), all of which fit the data equally well, i.e. are observationally equivalent. So that latter variation is not due to data.

3 Some examples of the SRC method

A simulated market model

Some data to work with was simulated from the following parameterized version of the market model

$$
\begin{aligned}
q_t &= -p_t + \eta_{1t} \quad (4)\\
q_t &= 3p_t + \sqrt{2}\eta_{2t} \quad (5)
\end{aligned}
$$

Equations (2) and (3) were then estimated with the SRC method using the simulated data from (4) and (5). In terms of the A, B construct used in EViews $A = \begin{bmatrix} 1 & -\alpha^* \\ 1 & NA \end{bmatrix}, B = \begin{bmatrix} NA & 0 \\ 0 & NA \end{bmatrix}$, where NA indicates that this term must be estimated, $\alpha^* = \frac{\theta}{(1-abs(\theta))}$, and θ comes from a $U(-1, 1)$ random number generator. 500 different values for α^* are generated, ML estimates of the remaining parameters in A, B are found, and impulse responses produced. So there are 500 sets of impulse responses that are to be either accepted or rejected. In fact, 15% of these are rejected. Of the retained impulse responses for quantity and price (with the demand shock first and costs second) we find that the closest fit to the true values of the impulse responses among the retained ones was[4]

$$SRC = \begin{bmatrix} .7369 & .3427 \\ .2484 & -.3605 \end{bmatrix}, True = \begin{bmatrix} .75 & .3536 \\ .25 & -.3536 \end{bmatrix}.$$

Consequently, it is clear that among the retained set of responses there is at least one that gives a good match to the true impulse responses. Changing the parameter values for simulating the market model data did not change this conclusion. Of course, if more than 500 draws for α^* are made, we would expect that eventually the true impulse response functions would be found among the generated ones.

[4] We just use a simple Euclidean norm to define the closest match to the true values. The impulse responses are for a one standard deviation shock.

In order to assess our method of generating α (called the basic method) it is worth looking at this example when the restrictions are placed on the coefficients rather than the impulse responses. Suppose that $\alpha > 0$ is assumed and that we estimate a demand elasticity δ with the data and a generated $\alpha^* > 0$. The two methods for generating α^* are our basic one, where $\alpha^* = \frac{\theta}{1-\theta}$ with θ coming from a U(0,1) density, and an alternative where α^* is generated from a U(0,600) density. Then Table 2 gives statistics on $\hat{\delta}^*$ for the cases where there are 1000 and 500000 draws (trials) of random numbers.

TABLE 2						
Comparing the $\hat{\delta}^*$ for two generating methods and number of draws						
		Basic draws=1000	*Alternative draws=1000*			
Median		-2.3	-.21			
Minimum		-13.06	-6.87			
Maximum		-.205	-.209			
		Basic draws=500000	*Alternative draws=500000*			
Median		-2.4	-.22			
Minimum		-13.08	-13.06			
Maximum		-.204	-.209			

It is clear that the statistics change little for our way of generating α while the alternative needs many trials to cover the space of α, resulting in the minimum changing quite a lot. Because $\hat{\delta}$ is a function of α for a given data set, and $\hat{\delta} < 0$ would be the only values retained in the draws, we can find the value α^* that gives the minimum value of $\hat{\delta}$ by just maximizing $\hat{\delta}(\alpha^*)'\hat{\delta}(\alpha^*)$. The maximum value can be found in a similar way. When this is done we get the values of -13.07 and -.204, so this suggests that our generation method is good at covering the $\alpha-$space at minimal cost. It is worth noting that the minimum and maximum values of δ come from *different models* i.e. different values of α are involved. Thus the minimum value comes from a model with a very inelastic supply and the maximum is from one that is moderately elastic.

A small macro model

A second example involves a small macro model with an output gap (y_{1t}), inflation (y_{2t}), and a policy interest rate (y_{3t}). We will write this as an SVAR with one lag ('SVAR(1)') in (6)-(8), although in the application it is an SVAR(2).

$$\begin{aligned} y_{1t} &= a^0_{12}y_{2t} + a^0_{13}y_{3t} + a^1_{12}y_{2t-1} + a^1_{13}y_{3t-1} + a^1_{11}y_{1t-1} + \varepsilon_{1t} & (6) \\ y_{2t} &= a^0_{21}y_{1t} + a^0_{23}y_{3t} + a^1_{22}y_{2t-1} + a^1_{23}y_{3t-1} + a^1_{21}y_{1t-1} + \varepsilon_{2t} & (7) \\ y_{3t} &= a^0_{31}y_{1t} + a^0_{32}y_{2t} + a^1_{32}y_{2t-1} + a^1_{33}y_{3t-1} + a^1_{31}y_{1t-1} + \varepsilon_{3t} & (8) \end{aligned}$$

In (6)-(8) the ε_{jt} are uncorrelated and have $E(\varepsilon_{jt}) = 0$, with standard deviations of σ_j. In contrast to the simulated data from the market model, actual data on the three variables taken from Cho and Moreono (2006). Sign restrictions for this small macro model were also studied in Fry and Pagan (2011), but done in a different way, and we will return to that in section 4. For reference purposes we will take the sign restrictions on the contemporaneous responses for *positive* shocks to be those in Table 3.

TABLE 3			
Sign restrictions for positive macro model shocks			
Variable\shock	*Demand*	*Cost-push*	*Interest rate*
y_t	+	-	-
π_t	+	+	-
i_t	+	+	+

To implement the SRC method we define

$$\begin{aligned} A_0 &= \begin{bmatrix} 1 & -a^0_{12} & -a^0_{13} \\ -a^0_{21} & 1 & -a^0_{23} \\ -a^0_{31} & -a^0_{32} & 1 \end{bmatrix}, A_1 = \begin{bmatrix} a^1_{11} & a^1_{12} & a^1_{13} \\ a^1_{21} & a^1_{22} & a^1_{23} \\ a^1_{31} & a^1_{32} & a^1_{33} \end{bmatrix} \\ B_0 &= \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \end{aligned}$$

In the model $n = 3$ so there are $\frac{n(n-1)}{2} = 3$ restrictions that are needed if one is to estimate the A_j matrices in a parametric way. Because it is assumed that there are no restrictions on A_1, the SRC method will proceed in the following way

(i) Generate $\bar{a}^0_{12}$, $\bar{a}^0_{13}$, $\bar{a}^0_{23}$ using $\bar{a}^0_{12} = \frac{\theta_1}{(1-abs(\theta_1))}, \bar{a}^0_{13} = \frac{\theta_2}{(1-abs(\theta_2))}$, $\bar{a}^0_{23} = \frac{\theta_3}{(1-abs(\theta_3))}$.

(ii) Apply MLE with A_0 containing these generated values (these are β_2) and estimate a^0_{21}, a^0_{31}, a^0_{32} (which are β_1) by ML.[5]

(iii) Form impulse responses using the resulting estimated values of A_0 and A_1, accepting or rejecting them based on the signs in Table 3.

Now in this case there are three random numbers that need to be generated - θ_1, θ_2 and θ_3. These will be independent and will all be drawn from U(-1,1). Again this is because we do not know which equation is the interest rate rule etc. so do not know the signs of coefficients.

As well as the ML approach we could use instrumental variables. Once a^0_{12}, a^0_{13}, a^0_{23} are fixed at $\bar{a}^0_{12}$, $\bar{a}^0_{13}, a^0_{23}$ the system is exactly identified, so IV and MLE are identical. The IV approach would involve the following steps:

(i) Construct $\bar{\varepsilon}_{1t} = y_{1t} - \bar{a}^0_{12}y_{2t} - \bar{a}^0_{13}y_{3t}$.

(ii) The dependent variable of (7) will be $y_{2t} - \bar{a}^0_{23}y_{3t}$. Using $\hat{\varepsilon}_{1t}$ as the instrument for y_{1t} an IV estimate of $\hat{a}^0_{21}$ can be found from (7), after which residuals $\bar{\varepsilon}_{2t} = y_{2t} - \bar{a}^0_{23}y_{3t} - \hat{a}^0_{21}y_{1t}$ can be computed.

(iii) Use $\bar{\varepsilon}_{1t}$ and $\bar{\varepsilon}_{2t}$ as instruments for y_{1t} and y_{2t} in (8) to estimate the remaining coefficients and hence complete the A_0 matrix

It is worth observing here that, rather than looking at impulse responses, one might use sign restrictions on the structural parameters, e.g. $a^0_{13} < 0, a^0_{21} > 0$ and $a^0_{32} > 0$. An even more complex restriction might be to ensure that the Taylor principle for stability held. Consequently, there are many types of restrictions that might be employed.

Unlike the market model with simulated data it is not easy to find impulse responses that satisfy the sign restrictions with the small macro model. Only around 5% of the impulse responses are retained. 1000 of these impulse responses are plotted in Figure 1. This is to match the equivalent figure in Fry and Pagan (2011) who used an alternative method of finding a range

[5] Of course the MLE must be done using a likelihood constructed so that the shocks are uncorrelated. But this is done in the SVAR options in packages like EViews and Stata.

of impulse responses to be discussed in Section 4.[6] It is clear that there is a large spread of values, i.e. many impulse responses can be found that preserve the sign information and which fit the data equally well. Note that the spread here is *across models* and has nothing to do with the variation due to the actual data.

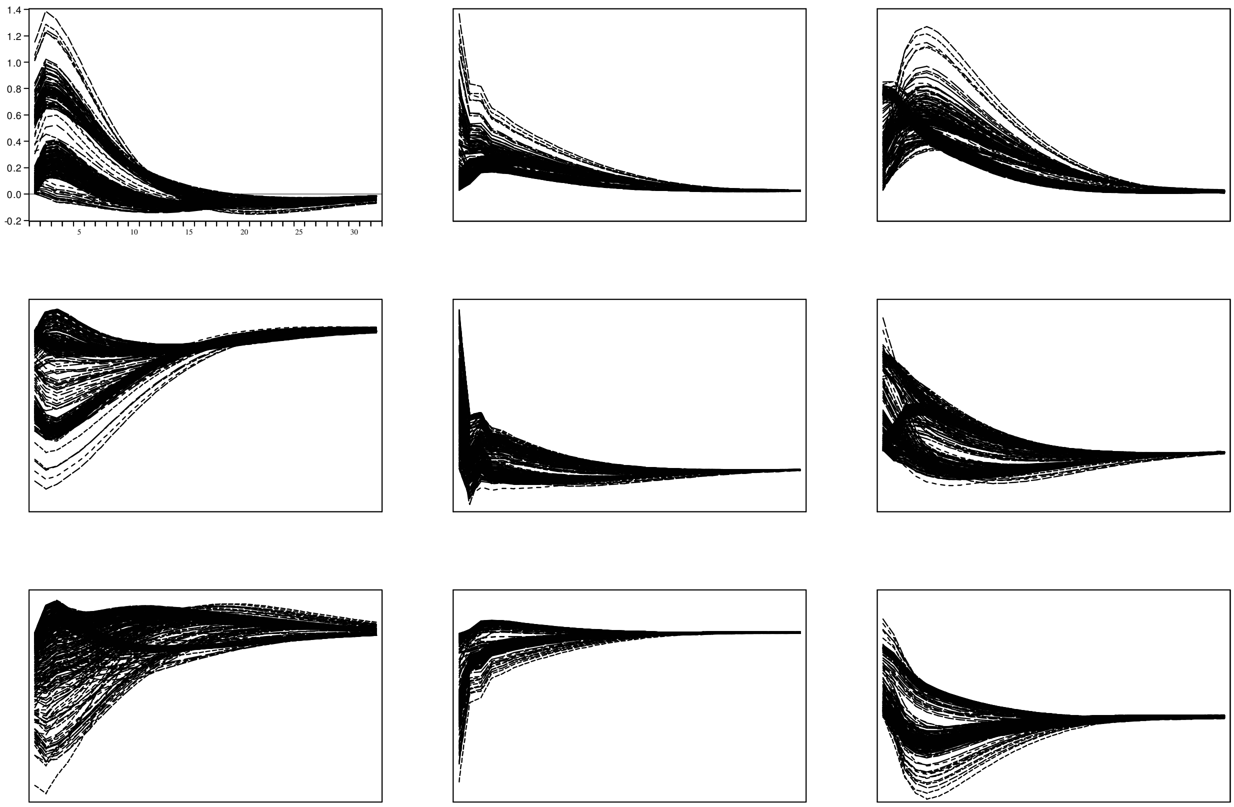

Figure 1: 1000 Impulses Responses from SRC Satisying the Sign Restrictions for the Small Macro Model using the Cho-Moreno Data

[6] In Figure 1 the positive cost shocks mean a negative productivity shock and, because, Fry and Pagan used a positive productivity shock in their figure, an allowance needs to be made for that when effecting a comparison.

Combining sign and short-run parametric restrictions on the small macro model

So far it has been assumed there are no plausible parametric restrictions. How does SRC proceed if there are such restrictions? To answer this it is first necessary to establish some extra notation. Thus the j periods ahead impulse responses of the n variables to the n structural shocks ε_t will be defined as C_j. For expository purposes now suppose that there is a SVAR(1). In this case the contemporaneous responses will be C_0 and others can be computed recursively using $C_j = A_1 C_{j-1}, j \geq 1$.[7] This SVAR would also have a VAR underlying it with the form $y_t = B_1 y_{t-1} + e_t$, and the impulse responses to the shocks e_t will be termed D_j, where D_j can be found from the recursion $D_j = B_1 D_{j-1}$ $(j \geq 1), D_0 = I_n$. Restrictions might also be applied to either C_j or $\sum_{k=1}^{\infty} C_k$. The first of these would be termed short-run and the second long-run restrictions.

A zero restriction on a contemporaneous impulse response

Suppose that in the small macro model the constraint is imposed that monetary policy has no contemporaneous impact upon the output gap. Letting the elements of C_0 be c_{ij}^0 this restriction would be $c_{13}^0 = 0$. The SRC method will now work with an SVAR in which this is imposed. Just as in previous sections, this could be done by either MLE or IV estimation, and we now look at both approaches in this context.

First, the MLE can be found using the (A, B) technology. In this structure $A_0 = I_n$, and $B_0 = \begin{bmatrix} b_{11}^0 & b_{12}^0 & 0 \\ b_{21}^0 & b_{22}^0 & b_{23}^0 \\ b_{31}^0 & b_{32}^0 & b_{33}^0 \end{bmatrix}$. It is clear that setting the (1,3) element of B_0 to zero imposes the parametric constraint, because $C_0 = A_0^{-1} B_0 = B_0$. As it stands however B cannot be estimated, as only a maximum of $\frac{n(n+1)}{2} = 6$ parameters can be estimated. Hence SRC would proceed by first generating values for b_{12}^0 and b_{23}^0 and then estimating the remaining elements of B_0 with MLE. After that impulse responses are found and either accepted or rejected. Thus the parametric restriction is automatically imposed before estimation by definition of B. Basically it is used to reduce the number of parameters to be estimated in B by one.

The alternative is to do IV upon the system $A_0 y_t = A_1 y_{t-1} + B\eta_t$, where B is diagonal with the standard errors of the shocks $\varepsilon_t (= B\eta_t)$. To do this

[7] There is always a recursion involving the SVAR lag coefficients for any order of the SVAR.

we would start with equation (8), where instruments would be needed for y_{1t} and y_{2t}. Now the VAR errors e_{jt} are linear combinations of the structural errors ε_{jt}. Pagan and Robertson (1998) pointed out that a restriction such as $c_{13}^0 = 0$ meant e_{1t} would be a combination of ε_{1t} and ε_{2t} alone, i.e. it would be uncorrelated with ε_{3t}. This means that the VAR residuals $\hat{e}_{1t}$ can be used as an instrument for y_{1t} in (8).

This points to the following strategy for estimation of the complete system by IV methods

(i) Generate values for a_{32}^0 and write the dependent variable of (8) as $y_{3t} - \bar{a}_{32}^0 y_{2t}$.

(ii) Estimate the resulting (8) with $\hat{e}_{1t}$ as the instrument for y_{1t} and thereby get residuals $\hat{\varepsilon}_{3t} = y_{3t} - \bar{a}_{32}^0 y_{2t} - \hat{a}_{31}^0 y_{1t}$. This can be used as an instrument in the other two equations.

(iii) Turning to (6) generate $\bar{a}_{12}^0$, set up the new dependent variable $y_{1t} - \bar{a}_{12}^0 y_{2t}$, and then estimate $\bar{a}_{13}^0$ using $\hat{\varepsilon}_{3t}$ as the instrument for y_{3t}. Then compute residuals $\hat{\varepsilon}_{2t}$.

(iv) Finally, estimate (7) by IV using the residuals $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$.

Once the two parameters are generated and the short-run restriction is imposed, the model is exactly identified so that these IV estimates are identical to the MLE.

A one-step ahead zero restriction on an impulse response

In this elements of C_1 are constrained to be zero. Now the system is set up with $A_0 = I$, $C_0 = B_0$ and $B_1 = A_1$, showing that

$$\begin{aligned} C_1 &= A_1 C_0 \\ &= B_1 C_0 \\ &= D_1 C_0. \end{aligned}$$

Because D_j can be computed from the VAR independently of any structural form it implies that restrictions on C_1 show up as linear restrictions upon the elements of $C_0 = B_0 = B$. This is easily handled in the (A, B) technology. Restrictions on higher order ($j'th$ lag) impulse responses will utilize $C_j = D_j C_0$. This result does not depend upon the expository device of making the SVAR of first order. McKibbin *et al.* (1998) used this approach when they were trying to find an SVAR representation for a calibrated macroeconomic model. They termed the resulting SVAR a hybrid model.

Combining sign and long-run parametric restrictions on the small macro model

An example is given here featuring the small macro model of Section 2 but now with a permanent shock. If there is to be a permanent shock in the system there must be at least one $I(1)$ variable and we will assume that this is the log level of GDP, calling it z_{1t}.[8] In SVARs such variables appear in differenced form, that is $y_{1t} = \Delta z_{1t}$. The SVAR system is then composed of y_{1t}, y_{2t} and y_{3t} with one permanent (supply) shock in the system, plus two transitory shocks associated with demand and an interest rate. By definition these transitory shocks have a zero long-run effect on output, z_{1t}. Before imposing any long-run restrictions the SVAR(1) system would be

$$\begin{aligned}
\Delta z_{1t} &= a_{12}^0 y_{2t} + a_{13}^0 y_{3t} + a_{12}^1 y_{2t-1} + a_{13}^1 y_{3t-1} + a_{11}^1 \Delta z_{1t-1} + \varepsilon_{1t} && (9)\\
y_{2t} &= a_{21}^0 \Delta z_{1t} + a_{23}^0 y_{3t} + a_{22}^1 y_{2t-1} + a_{23}^1 y_{3t-1} + a_{21}^1 \Delta z_{1t-1} + \varepsilon_{2t} && (10)\\
y_{3t} &= a_{31}^0 \Delta z_{1t} + a_{32}^0 y_{2t} + a_{32}^1 y_{2t-1} + a_{33}^1 y_{3t-1} + a_{31}^1 \Delta z_{1t-1} + \varepsilon_{3t}. && (11)
\end{aligned}$$

Now the two transitory shocks must have a zero long-run effect upon output, and we take these to be those attached to (10) and (11) i.e. ε_{2t} and ε_{3t}. Following Fisher *et al.* (2014) this restriction can be imposed on the system (9)-(11) by using the Shapiro and Watson (1988) approach of replacing (9) with

$$\Delta z_{1t} = a_{12}^0 \Delta y_{2t} + a_{13}^0 \Delta y_{3t} + a_{11}^1 \Delta z_{1t-1} + \varepsilon_{1t}. \qquad (12)$$

If the system (9)-(11) is written as a SVAR with matrices A_0 and A_1 there are now two restrictions between the elements of A_0 and A_1, namely $a_{12}^1 = -a_{12}^0$ and $a_{13}^1 = -a_{13}^0$. Hence the number of parameters to be estimated has been reduced by two through the use of the two long-run restrictions associated with the transitory shocks. In terms of the introduction, $K = 2$ and therefore it is possible to estimate $\frac{n(n-1)}{2} + K = 5$ parameters in the system (9), (10) and (12). In order to do this one of the unknown parameters must be generated, e.g. a_{23}^0. Once done ML estimation can be performed.[9]

[8] In Cho and Moreno the output gap was formed by regressing z_{1t} against a constant and a time trend and then using the residuals to measure it, so the underlying assumption was that z_{1t} was stationary around a deterministic trend. So here we are now taking it to be an $I(1)$ process with drift.

[9] There is a difficulty with getting EViews to do maximum likelihood as EViews does not allow for restrictions upon the A_1 matrix at the same time as enforcing uncorrelated shocks. Estimation can be done in EViews by instrumental variables.

To understand this better consider the IV approach. First (12) can be estimated by using y_{2t-1}, y_{3t-1} and Δz_{1t-1} as instruments. Once parameter estimates for (12) are obtained one can get residuals $\hat{\varepsilon}_{1t}$. In equation (10) a^0_{23} can be generated, producing a value $\bar{a}^0_{23}$, the equation can be re-arranged with $y_{2t} - \bar{a}^0_{23} y_{3t}$ as dependent variable, and then estimated using $\hat{\varepsilon}_{1t}$ as the instrument for y_{1t}. Finally, equation (11) is estimated using the residuals $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ as instruments. Again, once a^0_{23} is fixed the system is exactly identified and the IV estimator is the MLE. It is crucial to observe that, as $\bar{a}^0_{23}$ is varied, the long-run restrictions are always enforced by the *design of the SVAR,* i.e. by using (12) as part of it. Because these parametric (long-run) restrictions reduced the number of parameters to be estimated by two, only one parameter needs to be prescribed in order to get all the impulse responses. This contrasts with the three needed when all shocks were transitory. The role of sign restrictions is then to determine which of the two transitory shocks is demand and which is monetary policy. Because the permanent shock does not depend in any way upon the values assigned to a^0_{23}, it is invariant to the changing values of this coefficient, and so it remains the same. Estimating the SVAR with a permanent shock by the SRC technique now results in 45% of the responses satisfying all the sign restrictions, as compared to the 5% when all shocks were transitory and all variables were $I(0)$.

Modelling the effects of optimism shocks

Beaudry *et al.* (2011) investigated the role of news in fluctuations. There are five variables in their SVAR - stock prices, a measure of TFP, consumption, the real interest rate and hours worked (in this order). These will be labelled y_{jt} $(j = 1, .., 5)$ in that order. All shocks are assumed to be transitory so that the series are implicitly being treated as if they are $I(0)$. A VAR(4) is fitted.

Now one reason for using this example is to illustrate how our method works if only a single shock is to be identified in the system. In this case it is that attached to the structural equation for stock prices, termed an optimism shock in Arias et al. (2014). One parametric restriction is used, namely that the optimism shock has a zero contemporaneous effect on TFP. Otherwise the optimism shock is to be distinguished by using sign restriction information. They give three possible sets of restrictions and we will look at what they call "identification one". This says that the optimism shock has a positive effect on stock prices (as well as a zero contemporaneous effect on TFP). Basically the only difference between this case and the impact of a short-run restriction studied earlier for the small macro model is that the new system has five variables and only one shock is to be identified, i.e. there is only one structural equation of the form

$$y_{1t} = a^0_{12}y_{2t} + a^0_{13}y_{3t} + a^0_{14}y_{4t} + a^0_{15}y_{5t} + lags + b^0_{11}\eta_{1t},$$

where the η_{jt} are uncorrelated with unit variances (as in the introduction). To capture the VAR equations for y_{jt}, $j = 2, .., 5$ we therefore write them in the following way

$$y_{jt} = lags + b^0_{j1}\eta_{1t} + \sum_{k=2}^{j} b^0_{jk}\eta_{kt},\ j = 2, 3, 4, 5.$$

We use this form because the VAR equation errors must be allowed to be correlated with the structural error $\varepsilon_{1t} = b^0_{11}\eta_{1t}$, as well as between themselves. The coefficients b^0_{j1} ensures the former, while the common presence of terms like η_{2t} ensures the latter. Finally, the parameter b_{21} is set to zero to reflect the zero impact of optimism shocks on TFP. This system can then be placed into the (A, B) technology structure by defining

$$A = \begin{bmatrix} 1 & a^0_{12} & a^0_{13} & a^0_{14} & a^0_{15} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} b^0_{11} & 0 & 0 & 0 & 0 \\ 0 & b^0_{22} & 0 & 0 & 0 \\ b^0_{31} & b^0_{32} & b^0_{33} & 0 & 0 \\ b^0_{41} & b^0_{42} & b^0_{43} & b^0_{44} & 0 \\ b^0_{51} & b^0_{52} & b^0_{53} & b^0_{54} & b^0_{55} \end{bmatrix}$$

Now only $\frac{n(n+1)}{2} = 15$ parameters can be estimated in the system above, so that three of the a^0_{1j} must be generated and one can be estimated. Suppose the latter is a^0_{12}. This requires all the other a^0_{1j} to be generated, thereby producing values $\bar{a}^0_{13}$, $\bar{a}^0_{14}$ and $\bar{a}^0_{15}$. ML estimation can then proceed with the (A, B) structures described above. Instrumental variables works exactly as described with the small macro model, although in this case there is only one equation to estimate, and that will use $\bar{a}^0_{13}$, $\bar{a}^0_{14}$ and $\bar{a}^0_{15}$ along with residuals $\hat{e}_{2t}$ from the VAR equation for TFP as the instrument.

4 Alternative methods for using sign information on impulse responses

There is an existing method for using sign information on impulse responses in order to estimate SVARs. Because this method involves *recombination of an initial set of impulse responses* we will refer to it as SRR - sign restrictions by *recombination.* Hence, in this section we outline what SRR does and compare it to SRC. Fry and Pagan (2011) had a more detailed discussion about the logic of SRR in terms of the market model.

The SRR method

The key to the SSR method is to begin with a set of impulse responses for uncorrelated shocks that have unit variances. Given a VAR with errors e_t and $cov(e_t) = \Omega_R$, one can form uncorrelated shocks $v_t = Pe_t$, either by a Cholesky or a singular value decomposition (SVD). For the former $\Omega_R = A'A$, where A is a triangular matrix. Hence setting $P = (A')^{-1}$ produces uncorrelated shocks. These can then be transformed to shocks that have unit variances. The SVD expresses Ω_R as UFU', where $U'U = I, UU' = I$ and F is a diagonal matrix. Setting $P = U'$ will produce uncorrelated shocks with covariance matrix F and these can be transformed to have unit variances. Once these unit-variance shocks are found impulse responses to them can be computed, and this provides the initial set to be re-combined. Further impulse responses can be found by multiplying the original set by an $n \times n$ matrix Q that has the properties $Q'Q = I_n$ and $QQ' = I_n$. These properties are needed so as to ensure that the shocks remain uncorrelated with unit variances.

There have been a number of proposals about how Q should be generated. An early one was to use Givens matrices. For the $n = 2$ case the Givens matrix would be

$$Q = \begin{bmatrix} \cos\lambda & -\sin\lambda \\ \sin\lambda & \cos\lambda \end{bmatrix},$$

where λ lies between zero and π. As one uses different λ from this interval one gets different values for Q, and hence different impulse responses. One might use a random number generator to find a number of values for λ, drawing λ (say) from a uniform density over 0 to π. For every generated value of λ there will be a different model with different values for the impulse responses. All models are observationally equivalent, in that they produce an exact fit to the variance of the data on z_t.[10] Only those impulse responses producing shocks that agree with the maintained sign restrictions would then be retained. A more recent method for finding Q is described in Rubio-Ramirez et al. (2010), who use a simulation method for constructing a Q that has the requisite properties.

The Givens approach gets more complex when $n > 2$. In this case there are a number of Givens matrices, e.g. if $n = 3$ the matrix

$$Q_{12} = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

satisfies the necessary conditions. However, it is not unique, as there are two other matrices Q_{13}, Q_{23} formed by moving the rows of Q_{12} around to other positions given by the subscripts. Those using the Givens approach have taken the Q_{ij} to depend upon separate parameters λ_k $(k = 1, .., n)$ and then have worked with a product form like

$$Q_G(\lambda) = Q_{12}(\lambda_1) \times Q_{13}(\lambda_2) \times Q_{23}(\lambda_3).$$

Because the matrix Q_G above depends upon three different λ_k one could draw each λ_k from a $U(0,\pi)$ density function. As n grows the simulation approach in Rubio-Ramirez *et al.* (2010) is probably more computationally efficient.

Now, as was discussed earlier, the Q above cannot be used if there are either short-run or long-run parametric restrictions on the SVAR as well as sign restrictions on impulses. Ariel *et al.* (2014) have recently given an algorithm to construct Q such that the zero restrictions on the impulse responses are enforced through constraints on Q.

Comparing the SRC and SRR methods

The SRC method was a way of generating a range of impulse responses to uncorrelated shocks, so this is common to both SRR and SRC. It is worth thinking about SRR when a Givens matrix approach is used and when $n = 3$. Then three items need to be simulated - λ_1, λ_2 and λ_3- and these correspond

[10] This statement assumes a zero mean for z_t. It is worth observing that from equations (18) and (19) of Fry and Pagan (2011) the SRR method applied to the market model makes the structural parameters a function of the single parameter λ, and so selection of a value for this gives demand and supply curves. The mapping between the market model parameters and λ depends upon what model was chosen to initiate the process and the cosine and sine terms of the Givens matrix.

to the three θ_j used by the SRC method, except that λ are uniformly generated over $(0, \pi)$ and not over (-1,1). This equivalence remains for higher order n. Therefore the computational demands of both methods have some similarities, and problems arising from the dimensions of the system will be the same for both methods. It should be noted however that, when parametric restrictions are also applied along with sign restrictions, the number of θ_j may be much smaller, and this was seen in the earlier examples. Presumably this would also be the case with the restricted Q matrices that get produced by the Ariel *et al.* (2014) method.

So the difference between the methods comes down to how they perform in applications and their flexibility. To address the first we applied SRR to both the market model simulation data and the small macro model example. In relation to the first, the best fit to the true impulse responses in the 500 simulations was

$$SRC = \begin{bmatrix} .7369 & .3427 \\ .2484 & -.3605 \end{bmatrix}, SRR = \begin{bmatrix} .7648 & .3529 \\ .2472 & -.3563 \end{bmatrix}.$$

There seems to be a slightly better fit to the true values by SRC, although both methods work well. Turning to the small macro model, the SRR method produced a range of impulse responses that were presented in Fry and Pagan (2011). Comparing Figure 1 with the analogue (figure 1) in Fry and Pagan (2011) it seems as if SRC produces a broader range of impulse responses than SRR, e.g. the maximal contemporaneous effect of demand on output with SRC is more than twice what it was in Fry and Pagan (we emphasize that all impulse responses in figure 1 have the correct signs and they are all observationally equivalent). Whether this is an advantage or not is a moot point, as one might not be interested in extreme outcomes. It seems best to conclude that SRC has the potential to at least match what comes from SRR.

However, there are some extra advantages to SRC. First, as Pagan and Robertson (1998) observed in the context of a parametrically restricted SVAR, it is possible to get impulse responses with acceptable signs, even though the underlying structural equations have incorrect signs for their coefficients. In their case the IS curve had a negative real balance effect and the "correct" signs for impulse responses came from some cancellation. One can safeguard against this with SRC by imposing not only signs on the impulse responses but also upon the structural coefficients. This may also serve to narrow the range of impulse responses. Second, while the impulse responses produced by the SRR method is to one standard deviation shocks, the standard deviation is not estimated by the method. As Fry and Pagan (2011) pointed out, those using SRR often seemed to think that the responses were to one unit shocks, which is incorrect. Hence, knowing the standard deviation of the shocks would seem to be important for any policy discussions using sign-restricted impulses.

How then is it that the standard deviations can be estimated either when parametric restrictions or the SRC method are applied? The answer lies in the *normalization* used in those methods. Once this is provided the implied structural equations in the SRR method can be recovered, along with the standard deviations of their shocks. To illustrate, take the market model in (4)-(5), and write it in the form where A_0 is normalized to have unity on the diagonals and y_t has quantity and price in it. The demand and supply equations would then be

$$\begin{aligned} q_t &= -p_t + \eta_{1t} \\ p_t &= \frac{1}{3}q_t - \frac{\sqrt{2}}{3}\eta_{2t} \\ &= \frac{1}{3}q_t - .4714\eta_{2t} \end{aligned}$$

Taking the C_0 from SRR that was closest to the true value of the impulse responses for the market model, namely $C_0 = \begin{bmatrix} .7648 & .3529 \\ .2472 & -.3563 \end{bmatrix}$, gives $C_0^{-1} = \begin{bmatrix} .9905 & .9810 \\ .6872 & -2.1262 \end{bmatrix}$. Thereafter utilizing $A_0 = C_0^{-1}$ and imposing a normalization one gets the implied relations of

$$\begin{aligned} q_t &= -\frac{.9905}{.9810}p_t + \frac{1}{.9905}\eta_{1t} \\ p_t &= \frac{.6876}{2.1262}q_t - \frac{1}{2.1262}\eta_{2t} \\ &= .32q_t - .4703\eta_{2t}. \end{aligned}$$

From these equations the standard deviations of the shocks will be 1.01 and .4703 versus the true ones of 1 and .4714. Of course this means that, because many impulse responses are produced by SRR (and SRC), there will be many values for the standard deviations. Just as impulse responses need to be summarized in some way, this will be equally true of the standard deviations of the shocks found from the many models. There is not just one single standard deviation, unless a particular model is chosen by some criterion.

Some issues arising with sign restriction methods

It is worth observing that both SRC and SRR have a potential problem in generating the widest possible range of impulse responses. For SRR this arises in two ways. Firstly, in the selection of the initial set of impulse responses. As mentioned earlier, this has been done by either the Cholesky or singular value decompositions. The Cholesky decomposition requires an ordering of the variables, so there will be different initial impulse responses depending on which ordering one uses. Of course the SVD just adds another set. For any given Q then we would get a different set of impulse responses depending on which choice of factorization is used to initiate the process. Secondly, there is Q itself. The Givens and simulation based method provides a Q with the requisite properties, but there may well be others. If so, then one might expect different impulse responses when those Q matrices are applied to the same initial model.

This problem shows up with SRC as well. Now it is in terms of the parameters that are taken to be unidentified and which need to be generated. To be more concrete, consider the situation in section 3.2 where a_{12}^0, a_{13}^0 and a_{23}^0 were the generated parameters. Instead one might have chosen a_{31}^0, a_{32}^0 and a_{21}^0. If so, estimation would have started with (8) rather than (6).

For both methods this is a potential problem but perhaps not a real one (provided the number of trials is large). It may well be that the range of impulse responses generated is much the same, regardless of either the initial choice of impulse responses or the unidentified parameters. What might happen is that some choices require more trials than others in order to produce a relatively complete set of impulse responses. Fundamentally, the issue arises because both SRR and SRC focus on first producing a set of impulse responses to uncorrelated shocks, after which they can be checked to see if they satisfy sign restriction information. However, neither shows that this set is exhaustive.

It is worth observing that in section 3.5 there is only one structural equation estimated and so there are no other a_{ij}^0 then the ones generated there. This also occurs in the example of Gafarov and Olea (2015), who find maximum and minimum impulses by solving a quadratic programming problem, where the constraints are the sign restrictions and any zero restrictions on contemporaneous responses. Once they find the maxima and minima they use a delta method to get standard errors. Now maxima and minima are also found using our simulation method. When these are found we also know which models generated them i.e. the values of the generated coefficients, and can therefore find the standard errors for the responses directly from programs such as EViews, as it also uses a delta method. Our method has an advantage of working when there are long-run restrictions and zero restrictions on other lags than the contemporaneous ones. Consequently it seems to provide a relatively simple approach to doing the analysis that is in Gafarov and Olea. One problem in their approach occurs in their empirical example, where there is one shock and four variables. The maximum of the four impulse responses will come from different models, as is clear in our work. So, just like the problems identified with the median in Fry and Pagan (2011), one would need to find a single model that had four impulse responses that were as close as possible to the four maxima. One might proceed in the same way as the Median Target Method of Fry and Pagan, but now the target would be the impulse responses associated with the maxima.

Problems in using statistics such as the median also arise because a_{ij}^0 in SRC - and λ_j in SRR - are generated. As observed in section 3 this will mean that the distribution of impulse responses will depend on the way in which these quantities are generated. Hence quantities such as the median will change as the generation method changes. This point was made by Baumeister and Hamilton (2014) in their critique of Bayesian methods for summarizing the range of impulse responses. As Fry and Pagan (2011) pointed out, the median has little to recommend it - when data is generated from a model where the true impulse responses are known, they are often found at percentiles well away from the median. Indeed, that is evident from Table 2 where the true value of $\delta = -1$ was far from the medians found with both generating schemes. It seems less likely that the maxima and minima of impulse responses coming from a data set will be affected by different generating methods, and that was the case in the experiment recorded in Table 2. It is really the range of responses that one can get which is of ultimate interest.

5 Conclusion

The paper has outlined a method for handling sign restrictions in systems of simultaneous equations which are only partially identified. These sign restrictions might apply to either structural equation parameters or functions of them such as impulse responses. Initially a range of values for the unidentified parameters are generated and then the role of sign restrictions is to narrow the range. It is simple to apply and can be handled in packages such as EViews and Stata. Experiments show that it is no worse than existing methods and has some advantages - it applies to any simultaneous equations system and can incorporate a wider range of information e.g. on both the parameters and impulse responses. One potential application of the method is to micro-economic data sets, where parametric restrictions are often applied to produce estimates of items such as supply and demand elasticities, but it might be felt that weaker information such as signs would be more acceptable. Of course this comes at a cost in that there is no single estimate, but a range of values might be acceptable for many policy analysts, and in practice such scenarios are often considered.

6 References

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