Spectral Analysis using EViews

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Abstract

This paper explains how to perform univariate spectral analysis using an EViews addin. Suggestions to estimate the spectrum are explained across the article, specically for economic variables. All the add-in capabilities were applied using data of the Industrial production index, the exchange rate Euro/Dollar and the Chicago Board Options Exchange S&P 100 volatility index (VXO). Also, some tests for detecting periodic components, white noise and Gaussian white noise are included in the add-in. A procedure called Signicant Pass Filter (SPF) for extracting deterministic periodic components is showed; it has the advantage of eliminating the possibility of spurious cycles. Finally, a programing code to perform Dynamic Fourier Analysis (DFA) and estimate stochastic cycles using EViews is explained.

Keywords: DFA, EViews add-in, Spectral Analysis, SPF

1 Introduction

The analysis of time series is usually limited to the time domain analysis, ignoring the frequency domain perspective. Although both contain the same information, they differ in terms of presentation and interpretation e.g., is straightforward to find a deterministic wave that explain the series using spectral analysis, while it is not with the analysis in the time domain, therefore, they should be seen as complementary tools rather than competitive, [17, 8]. The purpose of this paper is to explain how to perform spectral analysis using EViews [12], from the theoretical and empirical point of view.

EViews is a statistical software for econometric and statistical analysis, it contains an easy to use point-and-clicking and a command line interface, powerful analytic tools and high quality graphs. Users also can write their own programs using the programing interface, for more details see [11]. This software works with the creation of objects i.e., to access the different statistical tools that EViews offers one must create a new EViews object in a workfile e.g., to apply a unit root test one must create a series object.

Since their 7.1 version EViews provides an add-in infrastructure, which allows the user to add new features that are indistinguishable from built-in ones i.e., new menu entries for point-and-clicking interaction or new command lines can be created by users. An EViews add-in will be explained across this paper [14].

The paper contain five sections including this introduction, sections 2 and 3 briefly mention the spectral estimators, their properties and suggestions for their estimate, in section 4 some spectral statistical test are explained and section 5 shows how to use the add-in with data of the industrial production index, the Chicago Board Options Exchange S&P 100 volatility index (VXO) and the exchange rate Euro/Dollar.

2 Discrete estimation

A second order stationary time series $x_t = x_1, ..., x_n$ can be written as a linear combination of orthogonal trigonometric functions[1], also known as the Fourier representation of finite sequences. If the angular frequency and n are known then:

$$x_t = \sum_{k=0}^{n/2} [a_k cos(\omega_k t) + b_k sin(\omega_k t)] \tag{1}$$

For $t = 1, 2, ..., n$. Where $\omega_k = 2\pi k/n$ and $k = 0, 1, ..., n/2$ are known as the Fourier frequencies, a_k and b_k as Fourier coefficients, see [9, 13, 17]. Equivalently the spectrum could be written as the Fourier representation of γ_k as:

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k e^{-iwk} \tag{2}$$

Where γ_k is the absolutely summable autocovariance function i.e., $\sum_{k=-\infty}^{\infty} \mid \gamma_k \mid < \infty$.

There are two main reasons for choosing the Fourier frequencies. First they allow the orthogonality between the trigonometric functions and the independence of the Fourier coefficients, which is of great convenience for hypothesis testing as will be seen in section 4. This independence holds asymptotically even with non-normality of x_t.

And second, as x_t is sampled at discrete points in time their spectral representation extends only to the interval $[-\pi, \pi]$, because we cannot distinguish between a frequency inside this interval to one outside it i.e., the components with frequencies $\omega + 2\pi, \omega - 2\pi, \omega + 4\pi, \omega - 4\pi, ...$ will appear to have frequency ω. Figure 1 shows two cycles generated with the cosine function and with angular frequencies $\omega = \frac{\pi t}{2}$ and $\omega = \frac{3\pi t}{2}$, the first being inside the interval and the second outside, empirically we only observe the coincidence points showed by the blue vertical lines, therefore the frequency $\omega = \frac{3\pi t}{2}$ is said to be an "alias" of $\omega = \frac{\pi t}{2}$. See [13].

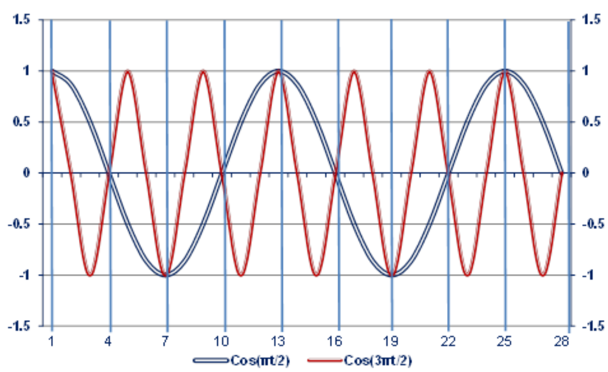

Figure 1: Aliasing.

This aliasing affects the ability to estimate the high frequency components in the spectrum, thus we miss detailed information of the rapidly oscillating component, to mitigate this effect the observations should be sampled with high periodicity.

There are three ways of estimating the spectrum of a time series, these are: the periodogram, the sample spectrum and the rational spectra. Since EViews already handles rational spectra this will be omitted, the periodogram and the sample spectrum will be explained below. It is worth noting that non of this methods are uniformly superior than the others, nevertheless if the population spectrum is discrete the rational spectra would not be the best choice, [13].

[1] For a brief introduction of the parameters in a trigonometric function see the appendix (on page 22)

2.1 Periodogram

The periodogram is a graph of the frequency in the abscissa axis[2] and the amplitude in the ordinates[3]. The estimators of the Fourier coefficients, following the Fourier representation in (1) are given by:

$$\hat{a_k} = \frac{1}{n}\sum_{t=1}^{n} x_t cos(\omega_k t) \tag{3}$$

$$\hat{a_k} = \frac{2}{n}\sum_{t=1}^{n} x_t cos(\omega_k t) \tag{4}$$

$$\hat{b_k} = \frac{2}{n}\sum_{t=1}^{n} x_t sin(\omega_k t) \tag{5}$$

Where (3) is used for $k = 0$ and $k = \frac{n}{2}$ if the number of observations n is even[4]. The expressions (4) and (5) are used for $k = 1, 2, ..., (n-1)/2$. The frequency ω_k is given by the Fourier frequencies.

2.2 Sample spectrum

The sample spectrum is based on the equation (2)[5], where their estimator is given by:

$$\hat{f}(\omega) = \frac{1}{2\pi}\sum_{k=-(n-1)}^{n-1} \hat{\gamma_k} e^{-iwk}$$

$$\hat{f}(\omega) = \frac{1}{2\pi}\left[\hat{\gamma_0} + 2\sum_{k=1}^{n-1}\hat{\gamma_k} cos(\omega k)\right] \tag{6}$$

The add-in programing of the discrete spectrum is based on the periodogram. It can be easily shown that the periodogram is related to the sample spectrum as $\hat{f}(\omega) = \frac{1}{4\pi}\hat{I}(\omega_k)$, see [17].

The periodogram and the sample spectrum are asymptotically unbiased estimators of the spectrum, nevertheless they are inconsistent in the sense that their variance does not decrease as the number of observations increase. This happens because the covariance between two different ordinates namely $cov(\hat{I}(\omega_1), \hat{I}(\omega_2))$ decreases as n increases, therefore the ordinates are independent and both, the periodogram and sample spectrum have an erratic form, [13].

3 Continuous estimate

If we know *a priori* that the spectrum is a continuous and smooth function then, estimating it based on the periodogram and the sample spectrum would not be the best approach. Nevertheless, using economic data one does not have this information, therefore the estimation must be performed using the discrete estimators and if the possibility of a continuous spectrum arises estimate it in a second step using continuous estimators.

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[2] In the add-in the frequency is presented as cycles per time unit, f in the appendix notation. It is also common to assign to this axis the angular frequency ω or k.

[3] The amplitude is given by $I(\omega_k) = \sqrt{\hat{a_k^2} + \hat{b_k^2}}$ for the kth frequency where $\hat{a_k}$ and $\hat{b_k}$ are the Fourier coefficients , see the appendix for details.

[4] In the add-in coefficient a_0 or mean is omitted, due to a distortion generated in the spectral density estimate based on the periodogram, see [17].

[5] Note that the autocovariance function could be recovered as: $\gamma_k = \int_{-\pi}^{\pi} f(\omega)e^{iwk}d\omega$. Therefore for $k = 0$ the variance is given by $\gamma_0 = \int_{-\pi}^{\pi} f(\omega)d\omega$, whereby the spectrum could be interpreted as the decomposition of the variance of the process or as the contribution of the component with frequency $(\omega, \omega + d\omega)$ to the variance of the process.

The estimator of the autocovariance function $\hat{\gamma_\tau} = \frac{1}{n}\sum_{t=1}^{n-|\tau|}(x_t - \overline{x})(x_{t+|\tau|} - \overline{x})$ is asymptotically unbiased and consistent, hence we may expect that a linear combination of this, as the sample spectrum, would be a consistent estimator of the same linear combination, [13]. However, as mentioned it is not, the reason is that the sample spectrum contains autocovariances for large τ, which is a poor estimator of the true autocovariance function since it is based in a small number of observations.

Thus an option of reducing the variance of the spectrum and obtaining consistency is to truncate the autocovariance function, therefore the expression in (6) will be given by:

$$\hat{f_w}(\omega) = \frac{1}{2\pi}\sum_{k=-M}^{M}\hat{\gamma_k} cos(\omega k) \tag{7}$$

Where M is the truncation point.

This will reduce the variance but also will increase the bias. Therefore a balance must be achieved, in general if we make M depends on N in such a way that $M \to \infty$ and $n \to \infty$, but M changes sufficiently slowly so that $\frac{M}{n} \to 0$ then the variance and the bias will tend to zero asymptotically. This estimator can be seen as a particular case of the more general form:

$$\hat{f_w}(\omega_k) = \frac{1}{2\pi}\sum_{k=-(n-1)}^{n-1}\lambda(k)\hat{\gamma_k} cos(wk) \tag{8}$$

Where for the truncated autocovariance function $\lambda(k) = 1$ for $\mid k \mid \leq M$ and 0 otherwise. The function $\lambda(s)$ is known as the lag window and as can be seen it is applied to the sample spectrum. On the other hand smoothing could be performed in the frequency domain i.e., applied directly to the periodogram as:

$$\hat{f_w}(\omega_k) = \sum_{j=-m_n}^{m_n} W_n(\omega_j)\hat{I}(\omega_k - \omega_j) \tag{9}$$

Where m_n is a function of the number of observations and $W_n(\omega_j)$ is known as the spectral window[6]. It can be proved that $W_n(\omega) = \frac{1}{2\pi}\sum_{k=-(n-1)}^{n-1}\lambda(k)e^{-ik\omega}$, that is, the spectral window is the Fourier transform of the lag window. Thus truncate the sample spectrum has the same effect than smoothing the periodogram.

For the add-in the spectrum was smoothed using spectral windows. To present the results it is only necessary to show the spectrum over the interval $[0, \pi]$ and since the spectral window contains n points the periodicity and the symmetric property of the spectrum were used to calculate the spectrum. The periodicity states that the spectrum has a period of 2π i.e., $f(\omega) = f(\omega + 2\pi)$ and symmetric states that $f(\omega) = f(-\omega)$, see [17]. Therefore from the computational point of view, the spectrum was ordered in the interval $[-\pi, 2\pi]$ to obtain the calculations using the spectral window.

There are different forms of the lag and spectral windows, for a summary of these see [16] and [13]. The add-in has the option of choosing seven spectral windows, these will be showed below.

In table 1 and in table 2 lists spectral and lag windows respectively. Is important to note that not all the lag windows are of the truncated type e.g., the Daniell and the Bartlett-Priestley, therefore M in their expression does not correspond to the truncation point, it corresponds to the degree of smoothing, more details are showed in the next section.

The Hamming and Hann windows are based on the general Tukey window with different parameter a[7], as can be seen these spectral windows are function of the Dirichlet kernel, denoted by $D_M(\omega)$ and it is equal to the truncated spectral window[8].

[6]This type of window must have the properties of $\sum_{j=-m_n}^{m_n} W_n(\omega_j) = 1$ and $lim_{n \longrightarrow \infty}\sum_{j=-m_n}^{m_n} W_n^2(\omega_j) = 0$.

[7]For the Hamming window $a = 0.23$ and for the Hann $a = 0.25$.

[8]In the progmming code the Hamming and Hann spectral windows are calculated replacing the definition of the Dirichlet kernel, see [17].

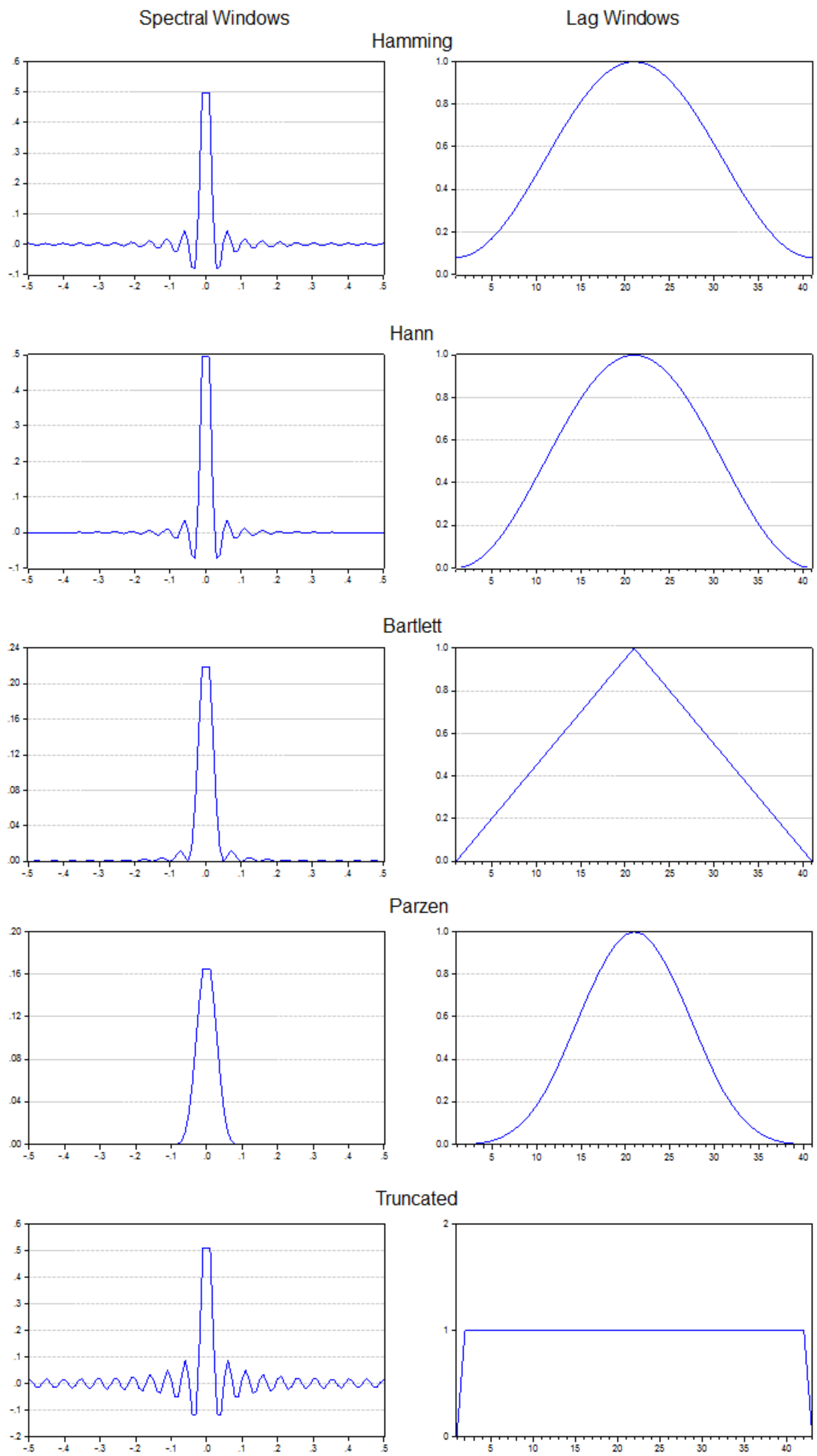

Figure 2: Spectral and lag windows.

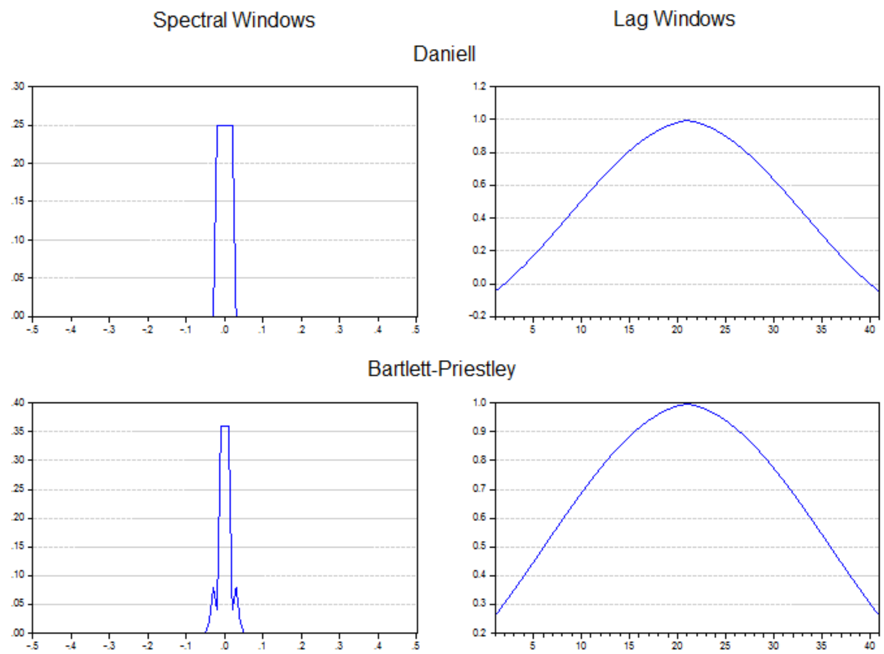

Figure 2: Spectral and lag windows.

Window name	Spectral window
Hamming	$W_n(\omega) = 0.23\left\{D(\omega - \frac{\pi}{M})\right\} + (1 - 2 * 0.23)D(\omega) + 0.23D(\omega + \frac{\pi}{M})$
Hann	$W_n(\omega) = 0.25\left\{D(\omega - \frac{\pi}{M})\right\} + (1 - 2 * 0.25)D(\omega) + 0.25D(\omega + \frac{\pi}{M})$
Bartlett	$W_n(\omega) = \frac{1}{2\pi M}\left\{sin(\frac{M\omega}{2})/sin(\frac{\omega}{2})\right\}^2$
Parzen	$W_n(\omega) = \frac{3}{8\pi M^3}\left\{sin(\frac{M\omega}{4})/\left[(\frac{1}{2})sin(\frac{\omega}{2})\right]\right\}\left\{1 - (\frac{2}{3})sin^2(\frac{\omega}{2})\right\}$
Truncated	$W_n(\omega) = \frac{1}{2\pi}\left\{sin\left[(M + \frac{1}{2})\omega\right]/sin(\frac{\omega}{2})\right\}$
Daniell	$W_n(\omega) = \frac{M}{2\pi}$ for $-\frac{\pi}{M} \leq \omega \leq \frac{\pi}{M}$ and 0 otherwise.
Bartlett-Priestley	$W_n(\omega) = \frac{3M}{4\pi}\left\{1 - (\frac{M\omega}{\pi})^2\right\}$ for $\mid \omega \mid\leq \frac{\pi}{M}$

Table 1: Spectral windows.

Window name	Lag window
Hamming	$\lambda(k) = 1 - 2 * 0.23 + 2 * 0.23cos(\frac{\pi k}{M})$ for $\mid k \mid\leq M$ and 0 otherwise
Hann	$\lambda(k) = 1 - 2 * 0.25 + 2 * 0.25cos(\frac{\pi k}{M})$ for $\mid k \mid\leq M$ and 0 otherwise
Bartlett	$\lambda(k) = 1 - \frac{\mid k \mid}{M}$ for $\mid k \mid\leq M$ and $\lambda(k) = 0$ for $\mid k \mid> M$
Parzen	$\lambda(k) = 1 - 6(\frac{K}{M})^2 + 6(\frac{\mid k \mid}{M})^3$ for $\mid k \mid\leq \frac{M}{2}$ and $\lambda(k) = 2(1 - \frac{\mid k \mid}{M})^3$ for $\frac{M}{2} \leq\mid k \mid\leq M$
Truncated	$\lambda(k) = 1$ for $\mid k \mid\leq M$ and $\lambda(k) = 0$ for $\mid k \mid> M$
Daniell	$\lambda(k) = sin(\frac{\pi k}{M})/(\frac{\pi k}{M})$
Bartlett-Priestley	$\lambda(k) = \frac{3M^2}{(\pi k)^2}\left\{sin(\frac{\pi k}{M})/(\frac{\pi k}{M}) - cos(\frac{\pi k}{M})\right\}$

Table 2: Lag windows.

Figure 2 shows the spectral and lag windows in the first and second column, both were calculated using $M = 20$.

The choice of the window is as important as the choice of the truncation point. It could be proved that the variance spectrum depends directly on the ratio $\frac{M}{n}$, the proportion of the used autocovariances and the bias depend inversely on the truncation point $\frac{1}{M}$, therefore as M decreases the variance decreases but the bias increases. The next sections are dedicated to the procedures of choosing a truncation point and a spectral window.

3.1 The spectral bandwidth

Before mentioning the methods of choosing a truncation point and a spectral window it is necessary to introduce the concept of spectral bandwidth. The bandwidth is defined as the distance between the half-power points ω_1 and ω_2, where the power or amplitude are defined by $f(\omega_1) = f(\omega_2) = \frac{1}{2}f(\omega_0)$, where ω_0 is the frequency of the maximum (minimum) value of the spectrum of the narrowest peak (trough). Figure 3 shows two spectra both with bandwidth $B_h = \omega_2 - \omega_1$.

An interesting feature is the relation of the spectral bandwidth and the rate of decay of the autocovariance function, the spectral bandwidth will be small if the autocovariance function decays slowly, and the bandwidth will be large if the the autocovariance decays rapidly

For example, an harmonic process with a single periodic component will have an non-decaying autocovariance function and a unique sharp peak in their spectrum, therefore their bandwidth will be small. On the other hand, consider a random white noise process, this will have a rapidly decaying autocovariance function and a flat spectrum, therefore their bandwidth are said to be infinite.

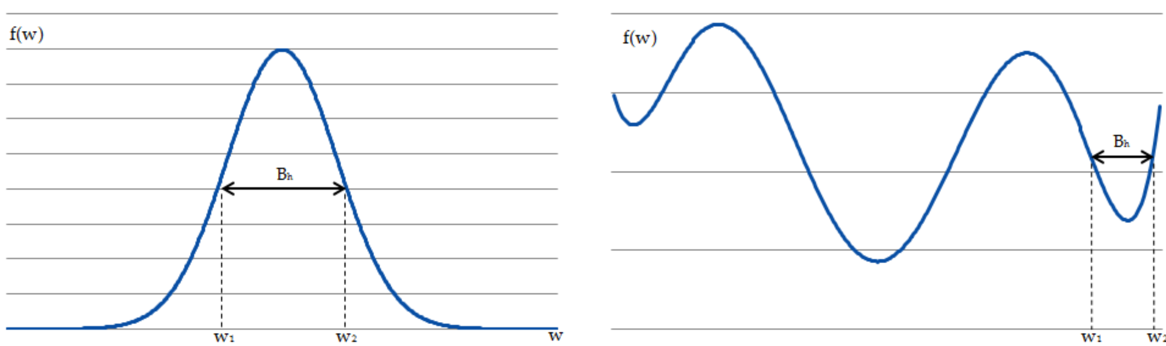

Figure 3: Spectral bandwidth.

3.2 Chossing a truncation point

The truncation point controls the decay rate of the lag window, or equivalently the width of the spectral window[9]. For example, if we generate the Bartlett spectral window for different truncation points, it can be seen that if the truncation point increases, the spectral window concentrates around the central frequency (the spectral window width decreases). This is showed in figure 4, were the window is plotted for truncation points of *10,20* and *30*.

[9]There are many definition of the width of a spectral window, but in general these are inversely related to the truncation point.

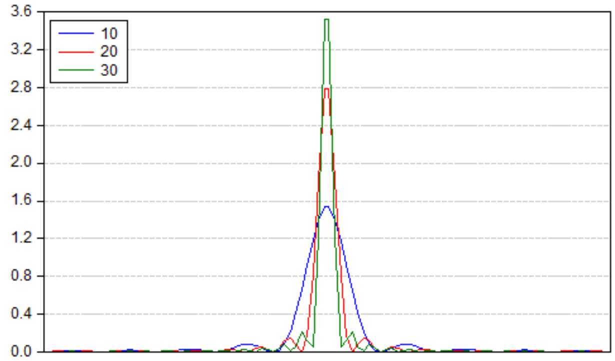

Figure 4: Bartlett-Bandwidth.

Now consider the population spectrum showed in figure 5, if a small enough truncation point is chosen so that $\frac{2\pi}{M} >\mid \omega_2 - \omega_1 \mid$ then the two peaks will be merged together. To resolve or separate the peaks one must choose M sufficiently large so that the width of the spectral window is not greater than the bandwidth of the narrowest peak of the spectrum. Therefore, to resolve the frequencies of the spectrum the following condition must be archived $width\ W_n(\omega) \leq B_h$, it would be advisable to leave some margin of safety and choose the width of the spectral window to be somewhat smaller than the bandwidth.

Another way to see this is that the spectral estimate will be resolved if their ordinates are uncorrelated[10], and this happens only if the spectral bandwidth is greater than the width of the spectral window.

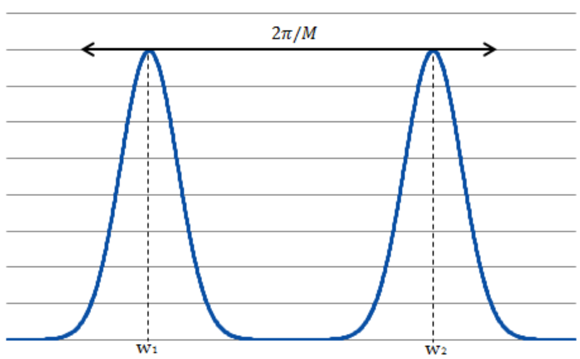

Figure 5: Window bandwidth.

As can be noted, to resolve frequencies it is necessary to have some prior information about the theoretical spectra, but in economic time series there is a lack of this information. Nevertheless, there are semi-empirical procedures which can be used, see [13].

- The autocovariance function: as mentioned, there is a relation between the spectral bandwidth and the rate of decay of the autocovariance (or autocorrelation) function, therefore a suggestion for choosing the truncation point is the value for which the autocovariance function is close to zero.

$$\hat{\gamma}(\tau) \sim 0, \mid s \mid < M \tag{10}$$

[10] It is important to note that in the spectral density estimator some correlation between frequencies were introduced in order minimize variance.

It should bear in mind that the autocovariance estimator decays more slowly than the theoretical function, given its autocorrelation, therefore the truncation could be overestimated. Also, if the the spectrum contains two peaks, one large and wide and other small and narrow, then the small peak will not be observed in the autocovariance function, hence the selected truncation will lead to an estimate without the small peak.

- Window closing: this method consists of choosing an arbitrary small truncation point an then compute a sequence of spectral estimates by closing the window i.e., increasing the truncation point, the initial estimates will be smooth, the procedure should stop until the smoothing has been relaxed too far. Another version of this method is to chose arbitrarily three truncation points one, small, medium and large, say M_1, M_2 and M_3 then try to find and intermediate value of M that should be chosen.
- M as a fixed proportion of N: this approach is based in that the ratio $\frac{M}{n}$ controls the variance. This method should take into account the properties of the underlying process.

3.3 Chossing a window

To choose a window there are two criteria that should be taken into account, the non-negativity property and the variance leakage, these will be explained below.

- Non-negativity: the choose of the window and the truncation point should not alter this property i.e., $f(\omega) =| f(\omega) |$. Table 3 summarize if a window satisfies this.

Window	Satisfies non-negativity
Hamming	No
Hann	No
Barlett	Yes
Parzen	Yes
Truncated	No
Daniell	Yes
Bartlett-Priestley	Yes

Table 3: Non-Negativity.

- Variance leakage: as can be seen from figure 2 some windows contain subsidiary side lobes, therefore this will give substantial contribution to the spectral density estimate if some peak coincides with some side lobe, hence it could introduce some bias. Nevertheless, this side lobes could be controlled with the truncation parameter, as with the Bartlett window showed in figure 4.

4 Tests

To explain the statistical tests for the spectrum it is necessary to known the sampling properties of it. If $x_t \sim N(0, \sigma_x^2)$ it can be showed that each Fourier coefficient a_k or b_k is $i.i.d$ $N(0, 2\sigma_x^2)$, where the independence property of the Fourier coefficients were used. Therefore, each periodogram ordinate has a distribution proportional to χ_2^2. Note that when n is even $I(0)$ and $I(\pi)$ has a distribution χ_1^2, see [17]. If x_t came from a linear process were its residuals are not necessarily Gaussian but $i.i.d$ then, the periodogram ordinates will be asymptotically normal by the central limit theorem.

4.1 Confidence intervals for spectral density

The distribution of the spectral density can be considered as a weighted linear combination of independent χ^2 variables, if the weights are not uniform then the distribution of the sum is no longer χ^2, nevertheless approximate distributions can be made, this have the form $\hat{f}_w(\omega_k) \sim \frac{f_w(\omega_k)\chi^2_\nu}{\nu}$ were ν are the equivalent degrees of freedom. In table 4 shows the parameter ν for the different windows used in the add-in.

Window	ν
Hamming	$2.5164n/M$
Hann	$8n/3M$
Barlett	$1.4n/M$
Parzen	$3.708614n/M$
Truncated	n/M
Daniell	$2n/M$
Bartlett-Priestley	$1.4n/M$

Table 4: Equivalent degrees of freedom.

Therefore the confidence interval at $(1-\alpha)100\%$[11] for $f_w(\omega_k)$ is given by:

$$\frac{\nu\hat{f}_w(\omega_k)}{\chi^2_{\alpha/2,\nu}} \leq f_w(\omega_k) \leq \frac{\nu\hat{f}_w(\omega_k)}{\chi^2_{1-\alpha/2,\nu}} \tag{11}$$

And for the logarithmic transformation of the spectrum[12] is given by:

$$ln\left[\hat{f}_w(\omega_k)\right] + ln\left[\frac{\nu}{\chi^2_{\alpha/2,\nu}}\right] \leq ln\left[f_w(\omega_k)\right] \leq ln\left[\hat{f}_w(\omega_k)\right] + ln\left[\frac{\nu}{\chi^2_{1-\alpha/2,\nu}}\right] \tag{12}$$

4.2 F test

This test is based on the periodogram and its objective is to find a hidden periodic component, with the hypothesis:

$$H0 : a_k = b_k = 0$$

$$Ha : a_k \neq b_k \neq 0$$

With the statistic[13]:

$$F = \frac{(a_k^2 + b_k^2)/\nu_1}{\sum_{j=1,j\neq k}^{n/2}(a_j^2 + b_j^2)/\nu_2} \sim F(\nu_1, \nu_2) \tag{13}$$

As mentioned the number of degrees of freedom of the periodogram ordinate depends on the tested frequency and if the number of observations is odd or even. Table 5 summarizes the values of ν_1 and ν_2 for n odd, even and different Fourier frequencies.

[11] The confidence intervals in the add-in are calculated at 95% of confidence.

[12] The logarithmic transformation is used for two reasons, first as a variance stabilizing technique, bringing the distribution of the statistic closer to normality and for a closer look of the spectrum.

[13] Neither the numerator or the denominator includes a_0.

	n odd	n even
$k = 1, ..., \frac{n}{2} - 1$	-	$\nu_1 = 2, \nu_2 = n - 3$
$k = 1, ..., \frac{n-1}{2}$	$\nu_1 = 2, \nu_2 = n - 3$	-
$k = \frac{n}{2}$	-	$\nu_1 = 1, \nu_2 = n - 2$

Table 5: Degrees of freedom.

For n odd, v_1 is always two since the test is not applicable for $\omega_k = 0$ ($k = 0$) and $\omega_k = \pi$ ($k = \frac{n}{2}$) and v_2 is always $n - 3$ since a_0, a_k and b_k are excluded from the sum. For n even, v_1 can be one as $b_{n/2} = 0$ for $\omega_{n/2} = \pi$ and two for the same reasons as for n odd; v_2 can be $n - 3$, due to a_0, a_k and b_k are excluded from the sum and $n - 2$ considering a_0 and $a_{n/2}$ are the only excluded.

4.3 Fisher and Whittle test

The Fisher test is designed to find only one hidden periodic component, and it is also based on the periodogram. Specifically it tests if the underling process is Gaussian white noise, in the sense that its maximum ordinate is not significant enough, i.e, $H0 : max\{I(\omega_k)\} = 0$. This test is based on the following statistic:

$$T = \frac{max\{I(\omega_k)\}}{\sum_{k=1}^{n/2} I(\omega_k)} \tag{14}$$

Under the null the critical values of the statistic can be calculated as:

$$P(T > g_\alpha) = \alpha \simeq \frac{n}{2}(1 - g_\alpha)^{\frac{1}{n/2-1}} \tag{15}$$

Where g_α is the critical value and α is the level of significance, in the add-in the significance is set at 5%.

The Whittle test extends the Fisher test to the second maximum ordinate of the periodogram, with the statistic:

$$T_2 = \frac{I^{(2)}(\omega_{(2)})}{\sum_{k=1}^{n/2} I(\omega_k) - I^{(1)}(\omega_{(1)})} \tag{16}$$

Where $I^{(i)}(\omega_{(i)})$ is the *ith* maximum. The procedure can be continued until an insignificant result is reached, therefore the statistic gives an estimation of the number of periodic components present in the series. The critical values of the statistic can be calculated as in (15), but $\frac{n}{2}$ must be replaced by $\frac{n}{2} - i + 1$ if the *ith* maximum is being tested.

This test works well if the true frequencies are close to the Fourier frequencies, if there are some different of the form $\frac{2\pi j}{N}$ then the power of the test will be certainly affected[14]. The power of the T_2 test to detect periodic component will be affected if these are of similar amplitudes, e.g., if a process contains two periodic components with small differences in amplitude then the T_2 statistic will be substantially smaller.

4.4 Normalized integrated spectrum

This test works with the hypothesis that the observations of the process come from a white noise process. It is based on the normalized integrated spectrum with the following statistic:

[14] There are two main procedures that could be performed to expand the range of frequencies with the consequence of losing the independence property, the first is expand the Fourier frequencies to $\omega = \frac{\pi k}{n}$ for $k = 1, ..., n$ or even $\omega = \frac{\pi k}{2n}$ for $k = 1, ..., 2n$ and finally secondary analysis, which is basically separate spectral analysis on observation groups, for details of the procedure and an empirical application see [13, 2].

$$U_p = \frac{\sum_{k=1}^{p} I(\omega_q)}{\sum_{k=1}^{n/2} I(\omega_q)} \tag{17}$$

Which is plotted against frequency and if the deviations of the statistic from the $\frac{p}{n/2}$ line do not exceed $\pm a\sqrt{2/n}$ the null will not be rejected, where a is set equal to 1.36 for the 95% of confidence. This test has the advantage that it is intensive from departures of normality, since it is based on the normalized integrated spectra.

5 Using the series object add-in

The spectral analysis add-in could be used from the point-and-clicking menu or from the command line. To use the add-in from the menu, or more specifically from a series object, the user has to click $proc \rightarrow add - ins \rightarrow spectral\ analysis$, this is showed in figure 6 in window one. In the second window there are options for choosing a spectral window, an output table with all the data, all the tests described in section 4, the periodogram components A and B and a logarithmic scale for the spectrum. Then two more windows will appear, these are used for the significant pass filter which will be explained below.

To use the add-in from the command line you have to follow the options showed in table 6. The data showed in the output table and the tests in the signal significant tests are subject to the user requested options e.g., if the user requests spectral density instead of the periodogram then the output table will contain spectral density data and the confidence intervals for the spectral density will be plotted.

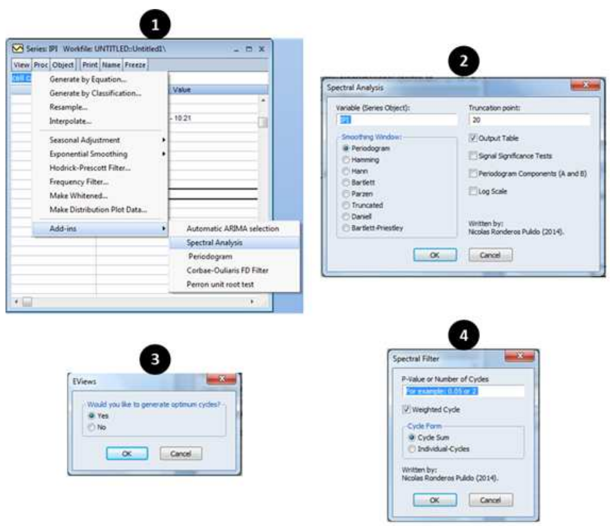

Figure 6: Menu.

series_name.spectral(options)	
Option	Command
Window	*Periodogram (default)*
	Hamming
	Hann
	Bartlett
	Parzen
	Truncated
	Daniell
	Bartlett-Priestley
Truncation	*truncation=number (20 default)*
Output table	*table*
Significance test signal	*t*
Periodogram components	*c*
Log scale	*log*
Significance signal test	Command option
Criteria for cycle selection	*criteria=probability or criteria=integer*
Non Weighted	*nw (weighted default)*
Individual cycles	*individual (cycle sum default)*

Table 6: Options for command line.

5.1 Some economic time series spectra

In this section some examples using the add-in will be showed, specifically it will be applied to the industrial production index (IPI), the Chicago Board Options Exchange S&P 100 volatility index (VXO) and the exchange rate Euro/Dollar with monthly data. These series are showed in figure 7 with their percentage changes.

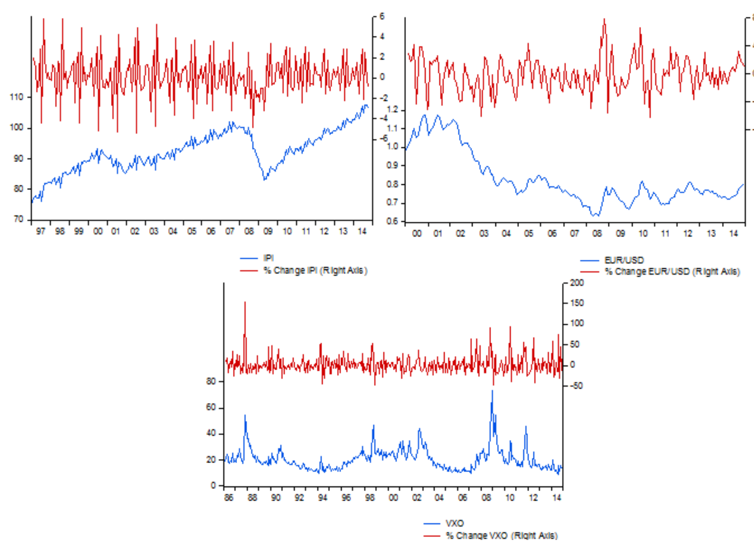

Figure 7: Time series data. Source: FED, FRED and Bloomberg.

The spectrum will be calculated for the percentage changes for the IPI and the EUR/USD, given the non-stationarity in mean of the series [15] which makes the autocovariance function not absolutely

[15] For the IPI the existence of a unit root was tested using the augmented Dikey-Fuller (ADF) test and Perron test

summable, the routine was called using *series_name.spectral(periodogram)* for each series. Figure 8 shows the periodogram for these. The transformation of the data has an effect on the form of the spectrum, this can be seen with the *transfer function* of the transformation, for the percentage change it is equal to $\mid \alpha(e^{i\omega}) \mid^2 = 2\left[1 - Cos(\omega_k)\right]$, therefore it preserves the high frequency (short-run) and eliminates the low frequency (long-run) component, like a high pass filter.

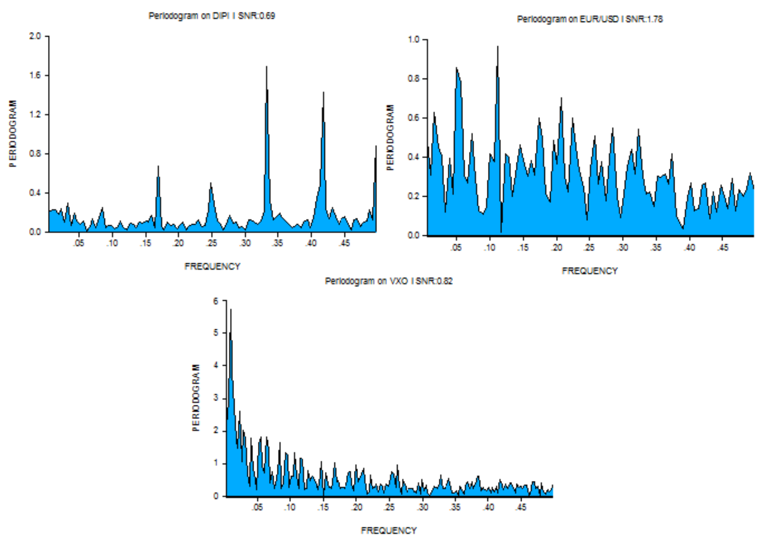

Figure 8: Spectrum of time series data.

The IPI periodogram suggest the presence of five deterministic cycles with frequency 0.17, 0.25, 0.33, 0.42 and 0.5 which correspond to a six, four, tree, two and a half and two monthly cycles, this can be attributed to seasonality. For the VXO spectrum it does not suggest the presence of a fixed cycle, instead it suggests the presence of a stochastic cycle or a pseudo-cyclical behavior, like in an autoregressive moving average (ARMA) model[16]. For the depreciation it is not clear if a fixed cycle exists, hence the spectra will be smoothed.

At the top of the spectral estimates there is a measure of precision of the estimation, known as the signal to noise ratio (SNR), this measure ignores the bias of the estimate and concentrates on the variance, an estimate with high SNR has more precision in the sense that its standard deviation is small compared with his mean.

$$SNR = \frac{E[\hat{f_w}(\omega)]}{var\left[\hat{f_w}(\omega)\right]^{1/2}} \tag{18}$$

The significance of the spectral components will be now tested using the tests described above, these are obtained using the command *series_name.spectral(periodogram,t)*. In figure 9 can be seen its results for the F, the Fisher and Whittle and the normalized integrated spectrum tests for the three series, the first column has the tests for $\Delta\%IPI$, the second for the depreciation and the third for the VXO. For the F test it is plotted the value of the statistical in (13) and their p value, therefore when the p value is close to zero the null of non-significance of the cycle with the respective frequency (or

with structural break in sample, there are enough evidence to support the existence of a unit root and a structural break. For the EUR/USD series the ADF test suggest the presence of a unit root. The ADF test was applied using the algorithm proposed by [5].

[16]It can be showed that the rational spectra of a ARMA process is of the form $f(\omega) = \frac{\sigma^2}{2\pi} \mid \frac{\phi_p(e^{-iw})}{\theta_q(e^{-iw})} \mid$ where for an AR(1) $f(\omega) = \frac{\sigma^2}{2\pi}\frac{1}{1+\phi^2-2cos\omega}$, therefore the VXO periodogram suggests a smooth spectra, like in a AR(1) with positive coefficient, see [17].

at least close to it) is rejected, for the Fisher and Whittle test is plotted the difference of the statistic given by (14) and (16) and the critical value g_α, hence if the value is positive the null of non-significance of the cycle (Gaussian white noise) is rejected and for the normalized integrated spectrum is plotted the statistical given by (17) and the deviation from the mentioned line, if the statistical exceeds one of the parallel lines the null of white noise is rejected.

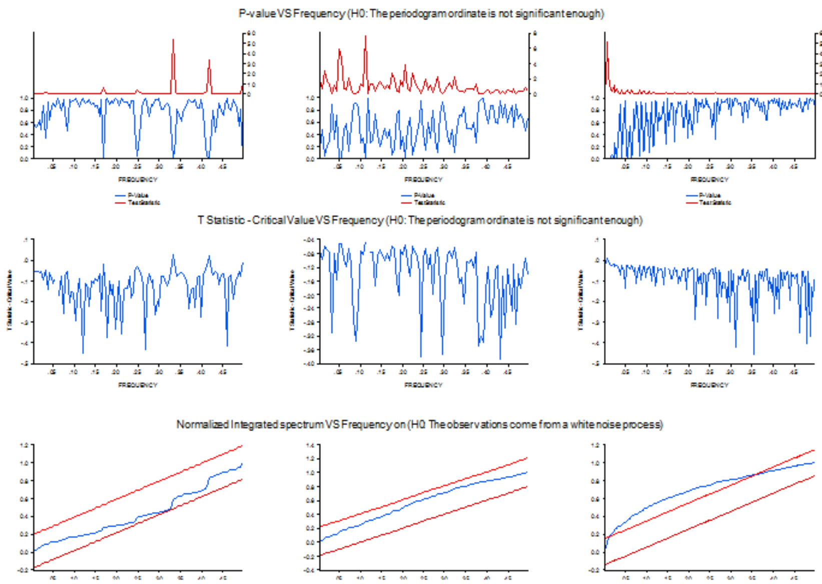

Figure 9: Significance signal tests.

The spectra of the depreciation and the VXO will be smoothed using the Bartlett-Priestley window with a truncation point selected by the autocovariance function method, for the depreciation the truncation was one i.e., *dtc.spectral(Bartlett-Priestley,truncation=1)* and for the VXO it was twenty i.e., *VXO.spectral(Bartlett-Priestley),* since its default is twenty. The two spectral density estimates are showed in figure 10.

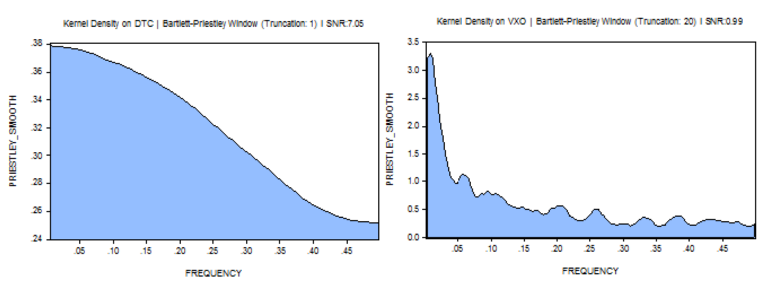

Figure 10: Spectral density.

As mentioned in the previous section, confidence intervals can be obtained when spectral density is requested, if the user chooses any of the spectral windows for estimation and the signal significance test then automatically the add-in will compute the CI, these are showed in figure 11 for the previous spectra. Note that the punctual estimator is not necessarily placed between the interval estimators, this occurs because the critical values were obtained from the χ^2 distribution.

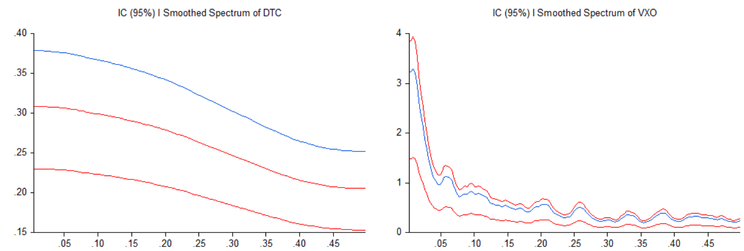

Figure 11: IC of spectral density.

5.2 Significant pass filter (SPF)

Usually the filters are applied in the time domain and try to extract a specific component of the series i.e., the long-run (smooth) or the short-run (rapidly and oscillating) component, their counterparty in spectral analysis would be the low frequency and high frequency components, respectively. A filter is said to be low-pass if it extracts the low frequency component[17], high-pass if it extracts the high frequency component and band-pass if the filter imposes a restriction to extract components in a band of frequencies. Examples of band-pass filters are the Baxter King and Christiano Fitzgerald filters, see [1] and [4].

The Fourier transform given in (1) can represent any discrete process in a complete way i.e., with no disturbance term. If these Fourier coefficients are used to forecast the process out of sample then the forecast will be exactly the same process, since the Fourier transform implies periodicity of the process. To assume that a economic time series is completely periodic is unreal, nevertheless some components of it could present this cyclical deterministic behavior, due to seasonality or business cycles. The purpose of the SPF is to find these deterministic components in all the spectrum, without imposing band restrictions.

The SPF is based on the form given in (1) were the filtered series is given by:

$$F_t = \sum_{k=0}^{n/2} [A_k cos(\omega_k t) + B_k sin(\omega_k t)] \tag{19}$$

Where $A_k = a_k$ and $B_k = b_k$ if the ordinate $I(\omega_k)$ is statistically significant or significantly large and if the weighted option is activated, if it is not activated then $A_k = 1$ and $B_k = 1$; if the ordinate is not significant then $A_k = 0$ and $B_k = 0$. In the add-in the statistical significance is obtained using the F test and a maximum level of tolerance for rejecting the null, given by a p value. For the significantly large criterion the kth maximum ordinates is used e.g., if this criterion is set to two, then the maximum and the second maximum ordinates will be used.

The filter assumes that if the kth ordinate satisfies the criterion then cycle will have a frequency ω_k of the form of the Fourier frequencies. Therefore it is assumed that the spectrum is discrete.

The SPF will be applied to the economic time series in the previous section using the biggest ordinate and the significant ordinate criteria i.e., an integer or a probability, for example the $\Delta\% IPI$ process could be recovered by the sum of 106 sinusoidal components, which covers all the spectra (since $n = 213$) using the command *dipi.spectral(periodogram,criteria=106),* notice they differ by a scale parameter which corresponds to the mean of the process, due to the exclusion of the parameters for $k = 0$.

[17]The moving average filter can be considered as low-pass filter, since its transfer function to the spectrum is of the form $\mid \alpha(e^{i\omega}) \mid^2 = \frac{1(1-cos(m\omega))}{m^2(1-cos(\omega))}$ where m is the number of averaged periods.

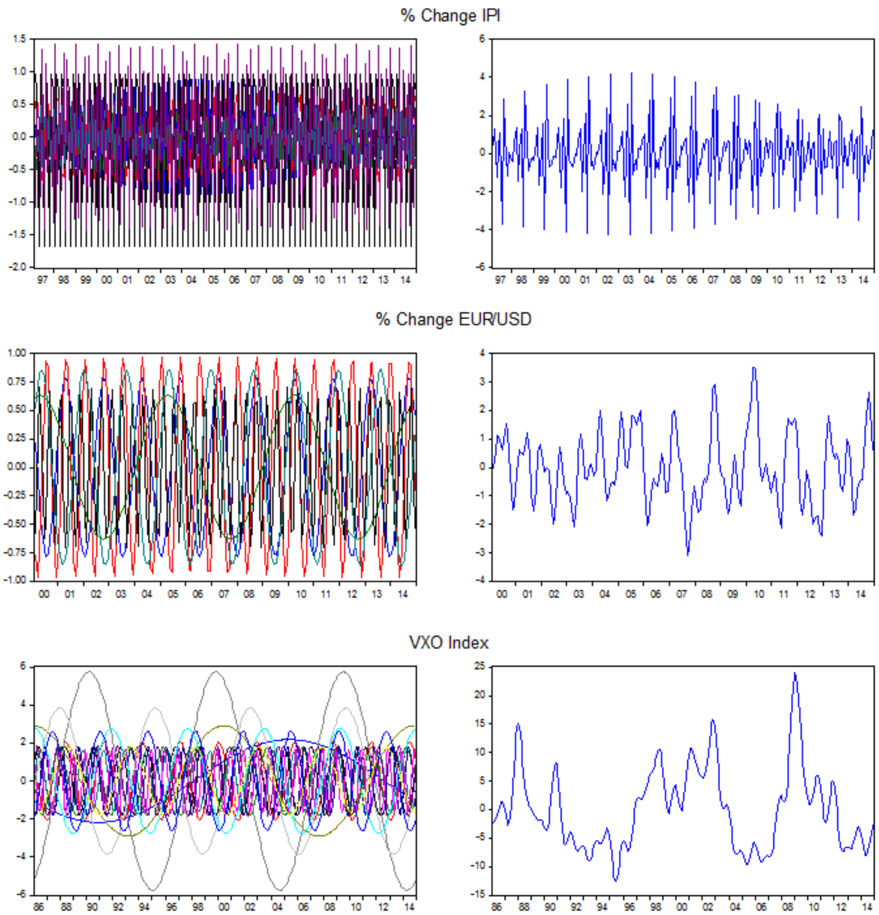

Figure 12: Filtered series.

To find only the underlying deterministic cyclical component in a series the user has two options, one is to find the individual cycle component of each series and add it manually or let the add-in add them automatically. Figure 12 shows the individual and the sum of the deterministic component of each series using a significance of 0.05 and weighted cycles, were the command *series_name.spectral(periodogram,criteria=0.05,individual)* is used to find the individual components and the individual option is excluded to calculate the sum of these.

It can be seen that the filtered series of the IPI presents a regular cyclical behavior, which can be attributed to seasonality, hence the SPF could be used as a method of seasonally adjustment by differencing the original series and the cyclical component. The filtered depreciation contains a smoother cyclical behavior, due to the relative large amplitudes of the low frequency component. The filtered series of the VXO index can be considered as a low-pass filter, since the only significant frequencies in their spectrum are low, therefore the filtered series is a smooth trend. To see exactly the frequency of the components contained in the filtered series the user can calculate the spectrum of the filtered series.

The SPF can also be used to estimate trends of integrated processes and used them to eliminate the non-stationary component e.g., figure 13 shows the observed and estimated trend of the exchange rate, their difference (observed-trend) and the spectrum of the difference, the trend was calculated using the SPF with the command *eurusd.spectral(periodogram,criteria=0.05)*. The spectrum of the difference series is exactly the same as the spectrum of the exchange rate in levels, except for the low-frequency component which generate the non-stationarity in the series. For the difference between the original and the trend series the unit root test ADF was applied, it rejects the presence of unit root.

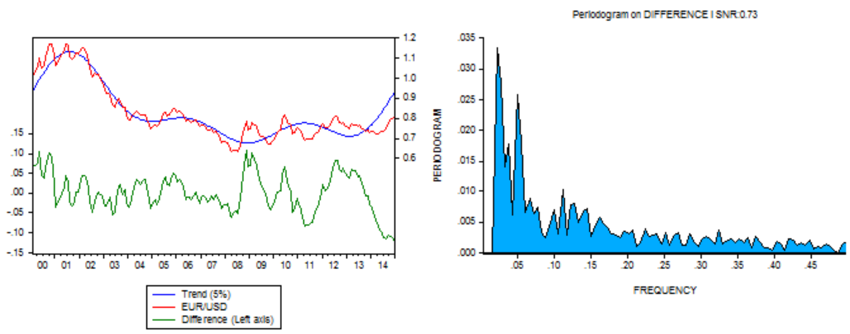

Figure 13: SPF trend.

The SPF has the advantage of eliminating the possibility of *spurious cycles*, which can arise due to linear trasnformations applied to the series with the objective of became it stationary, like differentiation or seasonal differentiation; another common linear transformation is the moving average, which is performed in several seasonal adjusting methods. This operations can also hide important attributes of the series, because it changes the relative importance of the cyclical components (gain) and the position of the series relative to time (phase). Therefore remaining the spectrum unchanged with stationarity does not generate spurious cycles and does not hide properties of the series, see [9].

The spectrum of an AR(1) with coefficient close to one in the limit tends to $f(\omega) = \frac{\sigma^2}{4\pi}\frac{1}{1-cos\omega}$, hence the significant frequency is always placed in the low frequency part of the spectrum. As mentioned, this type of process does not have an absolutely summable autocovariance function, therefore this procedure should be performed taken this into consideration, since strictly their spectrum does not exist.

It is important to note that the process could still have some information, that is, being autocorrelated after removing the SPF e.g., $x_t - F_t$ in an additive way, hence the SPF can be used repeatedly in this adjusted series if the filtered processes are independent[18]. For example, after seasonally adjust the industrial production, $\Delta\%IPI - F_t$, in this process still exists information to estimate the SPF, this iteration can be stopped until a white noise spectrum is reached i.e., a spectrum of the form $f(\omega) = \frac{\sigma^2}{2\pi}$, a flat spectrum.

Another use of the SPF is the use of the deterministic component for forecasting, given the deterministic nature of the filtered series this can be extended out of sample, to do this the user only has to expand the workfile range in EViews with the command *pagestruct(end=year:month)* and then calculate the SPF.

5.3 Dynamic Fourier analysis (DFA)

Fourier analysis can be extended beyond the estimation of the deterministic cycles of a second order stationary series to the estimation of stochastic cycles. Basically this stochastic cycles can be estimated by allowing the amplitude to vary over time i.e., $a_{k,t}$ and $b_{k,t}$, this is called DFA, see [15]. [9] developed a state-space estimation approach of stochastic cycles and a theoretical development of the pseudo-cyclical behavior described above.

A programming code of the DFA and the application of the SPF using the add-in will be explained, intuitively the code performs a time rolling estimation of the spectra. In figure 14 is showed the code.

The first line indicates the program to not send messages of calculation to the status line, lines 3 and 4 are inputs parameters of the step and the window, these are the window sample in which the

[18]It can be proved that the spectrum of the sum of two independent processes is the sum of the two spectra.

estimation is performed and the step at which the window moves, these numbers must be integers, in lines 6 to 10 are the series object name in the workfile for the calculation of their DFA, the spectral window[19] , the criteria selection for the dynamic SPF, log scale and the truncation point.

In lines 12 to 16 are defined the number of observations of the series, the position of the first and last observation of the series regarding the sample workfile, the number of iterations or rolls and a control variable to count the number of loops. From line 17 to 61 the estimation of the spectrum of one data window is perfomed, line 17 defines a control variable going from zero to the last possible complete data window moving with the step defined previously, then is the loop variable, in lines 19 to 21 is the sample for the workfile window, the first and last positions of the window are obtained transforming the position of the data to the date of the corresponding data, note that the window remains constant independently of the loop.

Lines 22 to 27 use the add-in to estimate the spectrum of the current sample window. Then, until line 32 a table called *dynamic* is created with the size given in brackets, it contains in the first row the sample page, which corresponds to the start and end of the current window, in the first column the frequency f is obtained from the output table of the add-in, then the estimated spectrum is placed in the $jth+1$ row and *!loop+1* column for the corresponding frequency and data window.

In lines 33 to 43 the SPF of the current data window is obtained, the first if statement is done to distinguish between the choosing criteria given by the user i.e., a probability or an integer number of maximum cycles, then the second if statement is made to locate the estimated filtered data, if this exists then it is positioned in a cycle vector in the same position that was located originally. Note that the cycle variable will be overwritten if the step parameter is lower than the data window, which is necessary to use all the information.

From lines 44 to 57 the output table and the estimated SPF are deleted, this is an important step since the next loops use the same objects names for the table and the SPF, hence if they were not removed the next loops could not obtain the information. Then, to line 60 a message is send to the status line displaying the completed rolling percentage. In line 61 the process is repeated, the first control variable is augmented by the step amount,the loop variable is increased by one, then the dynamic table is filled in the next column, the cycle vector is filled with new estimates from the SPF and the recent table and SPF are deleted until the control variable in the loop reaches its final value. Finally, the sample workfile is restored and a series object called *stochastic_cycle* is created.

This programming code is applied to $\Delta\%IPI$ and to the VXO. Unfortunately EViews does not handles graphs in three dimensions, hence the dynamic spectrum was plotted using Excel, figure 15 shows the results of the DFT and the stochastic cycle using the SPF. For $\Delta\%IPI$ the input parameters are showed in figure 14 and for VXO all the parameters were the same, except for the spectral window, were the Bartlett-Priestley was chosen with a truncation of twenty.

It can be seen that historically $\Delta\%IPI$ has presented fixed cycles of stable amplitudes at low frequencies, but decreasing at high frequencies, therefore the short-run cycles have provided relatively less to the variance of the process. For the VXO the lowest frequency component amplitude presents more variation, hence a stochastic cycle estimation is suggested.

[19]This could be any of the following options: Periodogram, Hamming, Hann, Bartlett, Parzen, Truncated, Daniell or Bartlett-Priestley.

```
mode quiet
'--------------------------------------------------------------Dynamic Spectra Inputs--------------------------------------------------------------
!step=1 'Step parameter for the rolling
!window=100 'Number of data that will be used to estimate the spectrum
'--------------------------------------------------------------Static Spectral Inputs--------------------------------------------------------------
%series="dipi"
%s_window="Periodogram" 'Periodogram, Hamming, Hann, Bartlett, Parzen, Truncated, Daniell, Bartlett-Priestley
!criteria=0.05 'Selection criteria for the spectral filter
%log_scale="N" 'Y/N
!truncation=1 'Truncation point
'----------------------------------------------------------------------------------------------------------------------------------------------------
!obs=@obs({%series})
!start=@ifirst({%series})
!end=@ilast({%series})
!nrolls=(!obs-!window)/!step
!loop=0
for !i=0 to !obs-!window step !step
    !loop=!loop+1
    %first = @otod(!start+!i)
    %last = @otod(!start+!i+!window)
    smpl {%first} {%last}
        if %log_scale="N" then
            {%series}.spectral({%s_window},table,truncation=!truncation,criteria=!criteria)
        else if %log_scale="Y" then
            {%series}.spectral({%s_window},table,truncation=!truncation,criteria=!criteria,log)
        endif
    endif
    for !j=1 to !window/2
        table(!window/2+1,!nrolls+2) dynamic(1,!loop+1)=@pagesmpl
        table(!window/2+1,!nrolls+2) dynamic(!j+1,1)=data(!j+1,2)
        table(!window/2+1,!nrolls+2) dynamic(!j+1,!loop+1)=data(!j+1,4)
    next
    for !j=!start to @obsrange
        if !criteria<1 then
            if sf_wsum(!j)<>NA then
                vector(@obsrange,1) cycle(!j,1)=sf_wsum(!j)
            endif
        else
            if mf_wsum(!j)<>NA then
                vector(@obsrange,1) cycle(!j,1)=mf_wsum(!j)
            endif
        endif
    next
    if !criteria<1 then
        %s_m="s"
        else
        %s_m="m"
    endif
    if %s_window="Periodogram" then
        delete data {%s_m}f_wsum {%s_window}01
    else if %s_window="Bartlett-Priestley" then
            %priestley=@right(%s_window,9)
            delete data periodogram_{%priestley} {%s_m}f_wsum
            else
            delete data periodogram_{%s_window} {%s_m}f_wsum
        endif
    endif
    scalar roll=@floor((!loop/!nrolls)*100)
    !roll_=roll
    statusline Rolling Percentaje: (!roll_%)
next
smpl @all
mtos(cycle,stochastic_cycle)
delete roll cycle
'----------------------------------------------------------------------------------------------------------------------------------------------------
```

Figure 14: Dynamic spectrum program

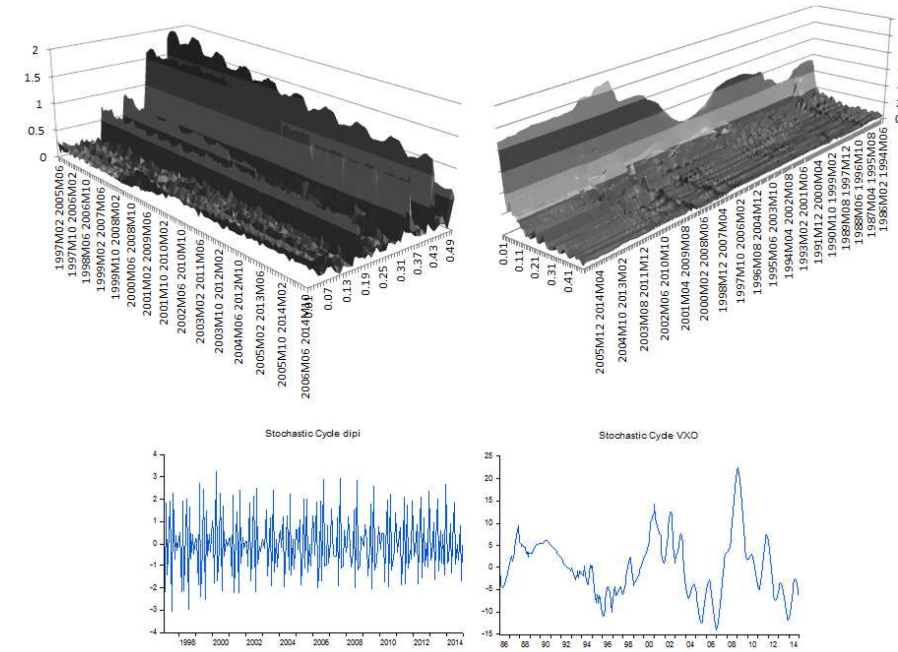

Figure 15: Dynamic Fourier transform-Stochastic cycle.

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Appendix

The trigonometric function:

$$y = \rho cos(\omega t - \theta)$$

Is defined in terms of the amplitude ρ, the angular frequency ω ,the phase angle θ and time t. The amplitude establishes the maximum and the minimum values of the cycle. The angular frequency can be written as $\omega = 2\pi f$, where f is the frequency given in cycles per time unit and $1/f$ is the duration of the cycle (or period) given in time unit per cycle. Any of these definitions of frequency serve to expand or contract y along the horizontal axis.

The phase angle gives the position of the function relative to the horizontal axis, and could be used to generate a shift of the cycle in radians, also the units of the phase angle could be turned in time terms i.e., $y = \rho cos(t - \xi)$ where $\xi = \theta/\omega$, since the function is a deterministic wave the same shift can be made by adding or subtracting a phase angle, this operations does not necessarily implies that the quantity added or subtracted should be the same.

Another way to introduce a shift in the cycle would be expressing the function as a sum of the cosine and sine functions as follows.

$$y = \alpha cos(\omega t) + \beta sin(\omega t)$$

where $\alpha = \rho cos(\theta)$ and $\beta = \rho sin(\theta)$, using the trigonometric identity $cos(x \pm y) = cos(x)cos(y) \mp sin(x)sin(y)$.

The procedure could be performed backwards to recover the shift and the amplitude using $\theta = tan^{-1}(\beta/\alpha)$ and $\rho^2 = \alpha^2 + \beta^2$ based in the trigonometric identity $cos^2(x) + sin^2(x) = 1$.