YapıKredi Invest

PAIRS TRADING MODEL

Mathematical Framework & Software Implementation

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What you should expect from the Model?

The model;

- **does not include any formal pair selection algorithm.**
- **requires pairs to be explicitly supplied.**
- **tests or verifies if the specified pairs are statistically valid.**
- **produces trading signals only for evaluation purposes.**
- **does not generate any kind of trading strategies or suggestions.**
- **aims to provide further insight into the pairs trading opportunities.**
- **should be supported by other assessment techniques.**
- **should only be considered as a decision support tool.**

Methods used in the Model

The model offers four different methods for the analysis of spread:

- **Normalized differences**
- **Cointegration**
- **Stochastic Spread**
- **Time Varying Mean Reversion**

And two different methods for the calculation of confidence bands:

- **Fixed**
- **Varying (i.e. moving average)**

Since all these methods are data dependent, the selection of period for the analysis is of crucial importance!

Normalized difference

Series are normalized as:

$$\widetilde{P}_t^i = \frac{P_{t-1}^i - E(P_{t-1}^i)}{\sigma(P_{t-1}^i)}$$

And then the spread is calculated as:

$$S_t = \widetilde{P}_t^1 - \widetilde{P}_t^2$$

The model produces a trading signal if:

$$S_t > \Delta \text{ or } S_t < \Delta$$

where Δ is a prespecified threshold value.

Cointegration

An error correction representation can be written as follows:

$$log(P_t^1) - log(P_{t-1}^1) = \propto_1 * \left(log(P_{t-1}^1) - \gamma * log(P_{t-1}^2)\right) + \varepsilon_t^1$$
$$log(P_t^2) - log(P_{t-1}^2) = \propto_2 * \left(log(P_{t-1}^1) - \gamma * log(P_{t-1}^2)\right) + \varepsilon_t^2$$

The spread can be considered as the long term equilibrium:

$$S_t = \left(log(P_{t-1}^1) - \gamma * log(P_{t-1}^2)\right)$$

The model produces a trading signal if:

$S_t > \Delta$ **or** $S_t < \Delta$

where Δ is a prespecified threshold value.

Stochastic Spread*

The model, as specified in Elliot et. al. (2005), is like the following:

$$S_t = x_t + D * \omega_t \qquad \omega_t \sim \mathrm{N}(0,1)$$
$$x_t = A + B * x_{t-1} + C * \varepsilon_t \qquad \varepsilon_t \sim \mathrm{N}(0,1)$$
$$A > 0, 0 < B < 1, C > 0, D > 0$$

The spread here is defined as:

$$S_t = \left(log\left(P_t^1\right) - log\left(P_t^2\right) \right)$$

The model produces a trading signal if:

$$S_t > \frac{A}{(1-B)} + \Delta * \frac{C}{\sqrt{2*(1-B)}} \text{ or } S_t < \frac{A}{(1-B)} - \Delta * \frac{C}{\sqrt{2*(1-B)}}$$ *(Ornstein-Uhlenbeck process)*

where Δ is a prespecified threshold value.

**** Robert J. Elliott, John Van Der Hoek & William P. Malcolm (2005): Pairs trading, Quantitative Finance, Vol. 5(3), pp. 271-276***

Time Varying Mean Reversion*

The model can be specified as follows:

$$S_t = A_t + B * S_{t-1} + D * \omega_t \qquad \omega_t \sim \mathrm{N}(0,1)$$
$$A_t = A_{t-1} + dA_{t-1}$$
$$dA_t = dA_{t-1} + C * \varepsilon_t \qquad \varepsilon_t \sim \mathrm{N}(0,1)$$
$$0 < B < 1, C > 0, D > 0$$

The spread here is defined as:

$$S_t = \left(log\left(P_t^1\right) - log\left(P_t^2\right)\right)$$

The model produces a trading signal if:

$$S_t > \frac{(\hat{A}_t | S_{t-1})}{(1-B)} + \Delta * \frac{E(\hat{\sigma}_t(\hat{A}_t | S_{t-1}))}{(1-B)} \text{ or } S_t < \frac{(\hat{A}_t | S_{t-1})}{(1-B)} - \Delta * \frac{E(\hat{\sigma}_t(\hat{A}_t | S_{t-1}))}{(1-B)}$$

where Δ is a prespecified threshold value.

** Eren Ocakverdi (2011): A Simple Pairs Trading Strategy for ISE30, Yapi Kredi Occasional Macro Notes, May 2, 2011.*

Additional Output

Gregory-Hansen procedure[1] is a formal test that investigates the existence of cointegration relationship in the presence of an unknown structural break, which is endogenously identified.

Dynamic conditional correlation[2] is a nonlinear model of time varying correlation within a multivariate framework.

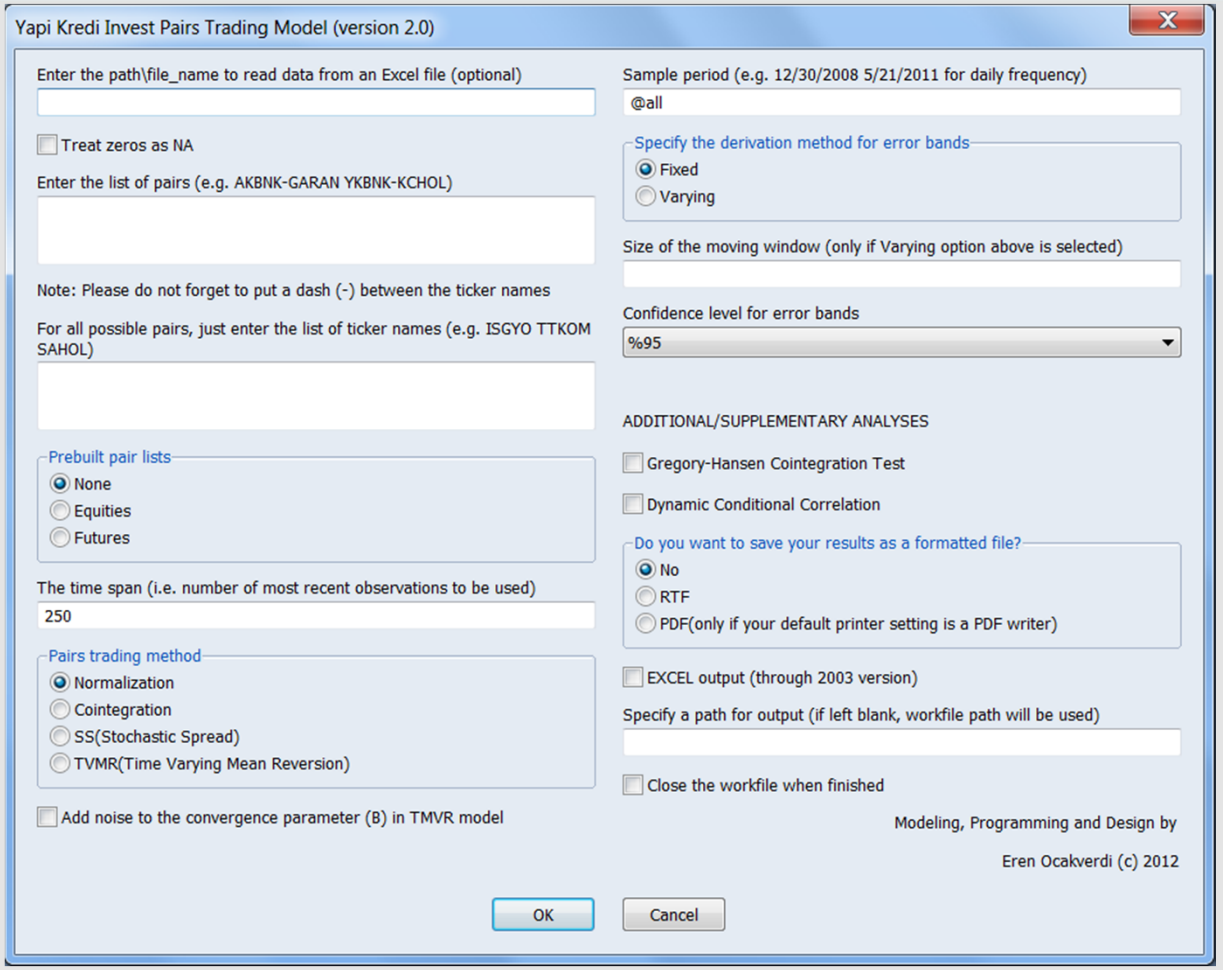
[1] *Gregory, A. W. and Hansen, B. E. (1996). "Residual-Based Tests for Cointegration in Models with Regime Shifts", Journal of Econometrics, Vol. 70, pp. 99-126.*

[2] *Engle, R. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models," Journal of Business & Economic Statistics, American Statistical Association, Vol. 20(3), pp. 339-50*

Software Implementation

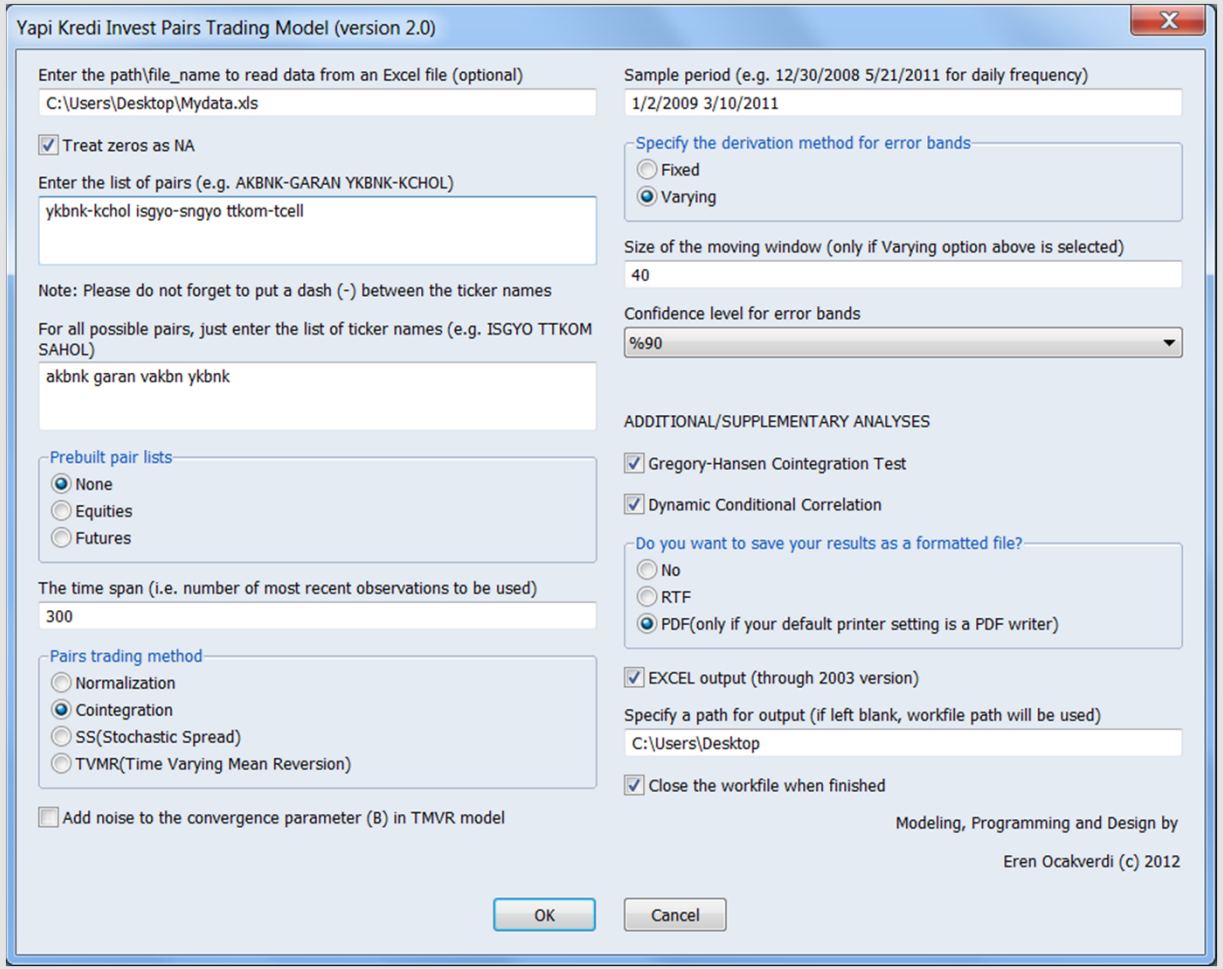
- Heavy computational burden of the methods used requires a specialized software.
- The software should have advanced statistical and/or econometric features.
- Ability to handle complex data structures and to easily exchange data with other programs (e.g. Excel) and databases is important.
- We prefer to use EViews, since it also has an Add-in infrastructure that provides easy access to user-defined programs.
- All methods are built and analyzed via EViews' estimation and/or programming features.
- An Add-in is developed to allow the user to quickly perform and repeat all the analyses.

Add-in Interface*



**** EViews version 7.1 or later is required to install and run the Add-in.***

Example

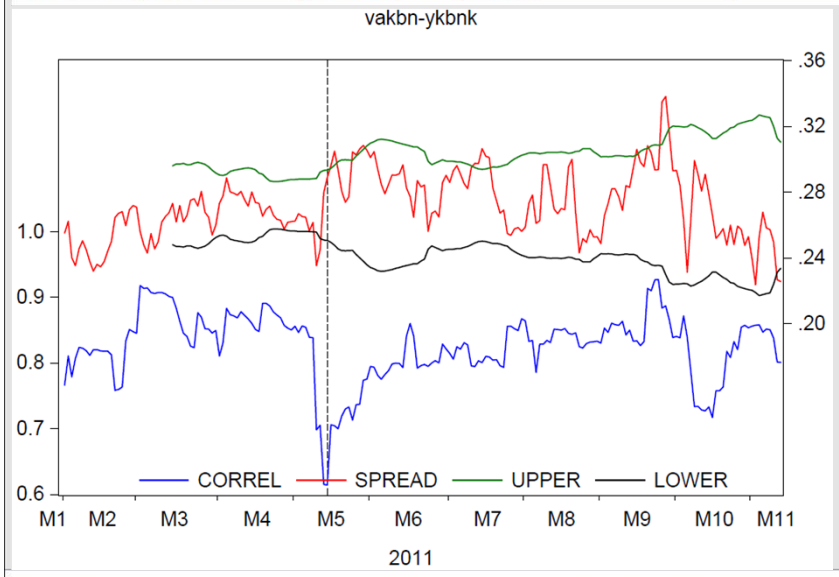


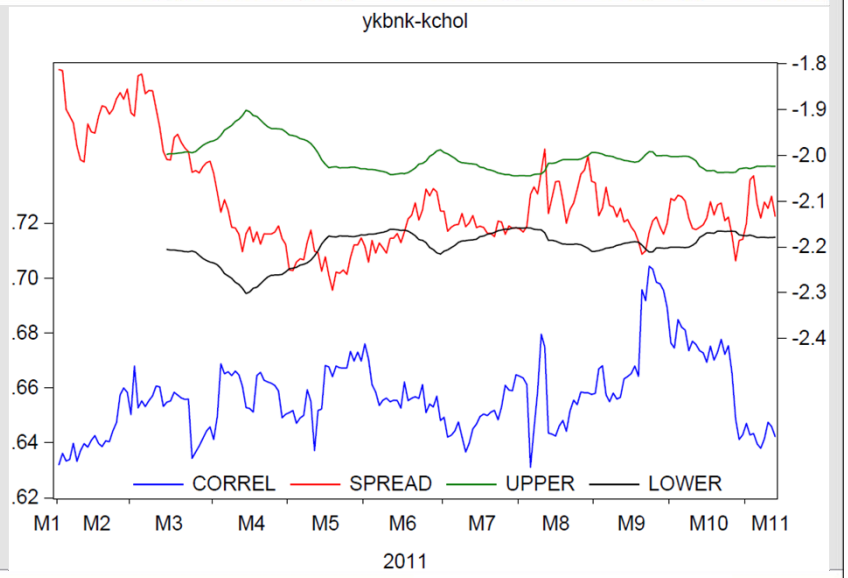
Note: The Add-in can be run with default settings as long as valid pairs are supplied.

Sample Output (e.g. cointegration)

TRADING_SIGNALS

	Long	Short	Hold	Message-1	Message-2
ykbnk-kchol			X	Pair is not cointegrated	log(YKBNK(-1)) - 1.816*log(KCHOL(-1)) + 2.102
isgyo-sngyo			X		log(ISGYO(-1)) - 0.694*log(SNGYO(-1)) + 0.064
ttkom-tcell			X	Pair is not cointegrated	log(TTKOM(-1)) + 1.265*log(TCELL(-1)) - 4.754
akbnk-garan			X		log(AKBNK(-1)) - 1.147*log(GARAN(-1)) + 0.265
akbnk-vakbn			X		log(AKBNK(-1)) - 0.784*log(VAKBN(-1)) - 0.971
akbnk-ykbnk			X	Pair is not cointegrated	log(AKBNK(-1)) - 0.555*log(YKBNK(-1)) - 1.198
garan-vakbn			X	Pair is not cointegrated	log(GARAN(-1)) - 0.661*log(VAKBN(-1)) - 1.106
garan-ykbnk			X	Pair is not cointegrated	log(GARAN(-1)) - 0.463*log(YKBNK(-1)) - 1.305
vakbn-ykbnk	vakbn	ykbnk			log(VAKBN(-1)) - 0.721*log(YKBNK(-1)) - 0.272





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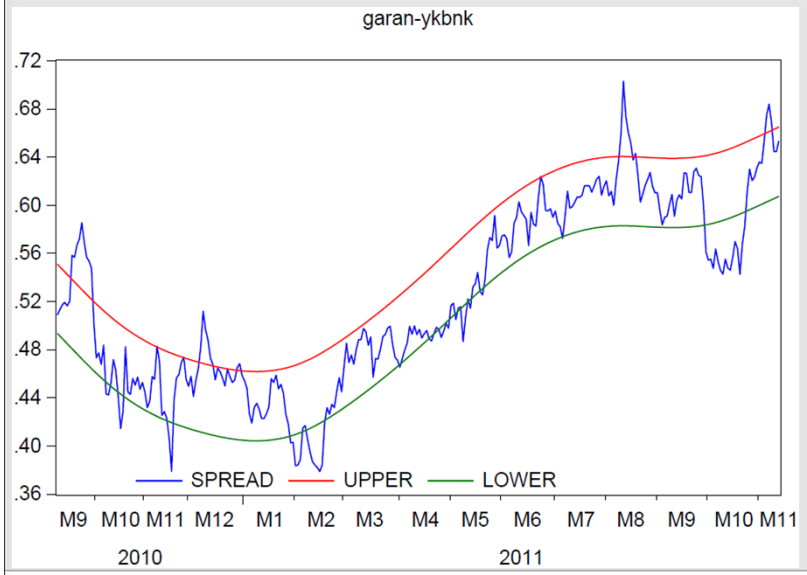
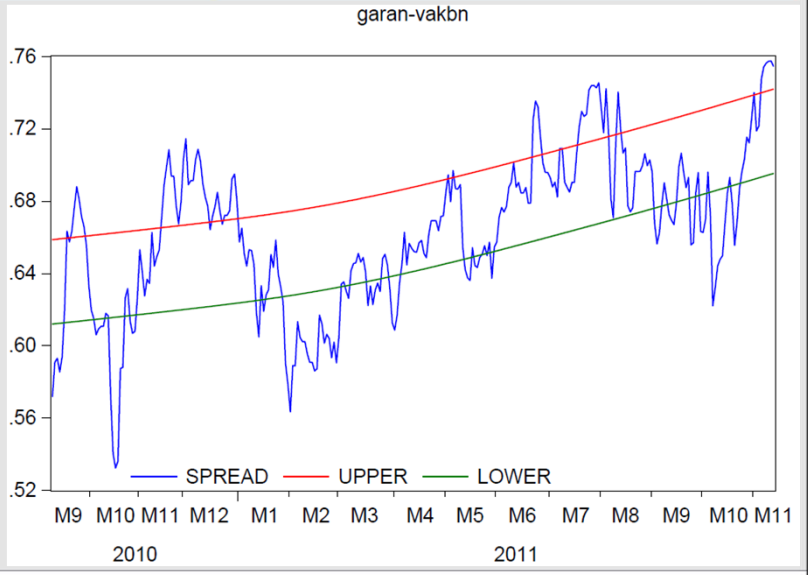
Sample Output (e.g. time varying mean reversion)

TRADING_SIGNALS

	Long	Short	Hold	Message-1	Message-2
ykbnk-kchol			X		Convergence achieved after 30 iterations
isgyo-sngyo			X	WARNING: Singular covariance	Convergence achieved after 19 iterations
ttkom-tcell			X		Convergence not achieved after 1000
akbnk-garan			X	WARNING: Singular covariance	Convergence achieved after 27 iterations
akbnk-vakbn	vakbn	akbnk		WARNING: Singular covariance	Convergence achieved after 15 iterations
akbnk-ykbnk			X	WARNING: Singular covariance	Convergence achieved after 24 iterations
garan-vakbn	vakbn	garan			Convergence achieved after 20 iterations
garan-ykbnk			X		Convergence achieved after 22 iterations
vakbn-ykbnk	vakbn	ykbnk		WARNING: Singular covariance	Convergence achieved after 25 iterations





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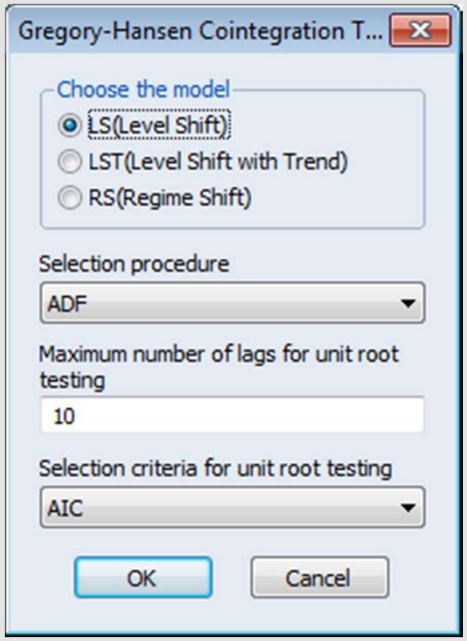
Sample Output (e.g. Gregory-Hansen Test)



LEVEL SHIFT	t-stat	Lag	Break
ykbnk-kchol	-5.602585	1.000000	3/28/2011
isgyo-sngyo	-5.010057	0.000000	1/27/2011
ttkom-tcell	-3.968144	7.000000	4/18/2011
akbnk-garan	-4.233318	0.000000	6/28/2011
akbnk-vakbn	-4.237000	0.000000	1/20/2011
akbnk-ykbnk	-4.889521	0.000000	3/30/2011
garan-vakbn	-4.118853	1.000000	1/14/2011
garan-ykbnk	-3.960051	0.000000	4/07/2011
vakbn-ykbnk	-6.049797	1.000000	2/24/2011

LEVEL SHIFT with TREND	Za-stat	Za-break	Zt-stat	Zt-break
ykbnk-kchol	-49.21068	9/08/2011	-5.449845	4/04/2011
isgyo-sngyo	-45.45040	7/25/2011	-5.003459	6/27/2011
ttkom-tcell	-25.96880	7/20/2011	-3.620100	7/20/2011
akbnk-garan	-32.54835	6/23/2011	-4.187868	6/23/2011
akbnk-vakbn	-38.37753	1/14/2011	-4.472096	1/14/2011
akbnk-ykbnk	-45.97391	3/29/2011	-5.140182	3/30/2011
garan-vakbn	-37.25759	1/13/2011	-4.479971	1/13/2011
garan-ykbnk	-31.07020	4/08/2011	-4.119654	5/17/2011
vakbn-ykbnk	-69.96113	2/18/2011	-6.805542	2/22/2011

Significant (highlighted) break points are also drawn in the related graphs as dashed lines.

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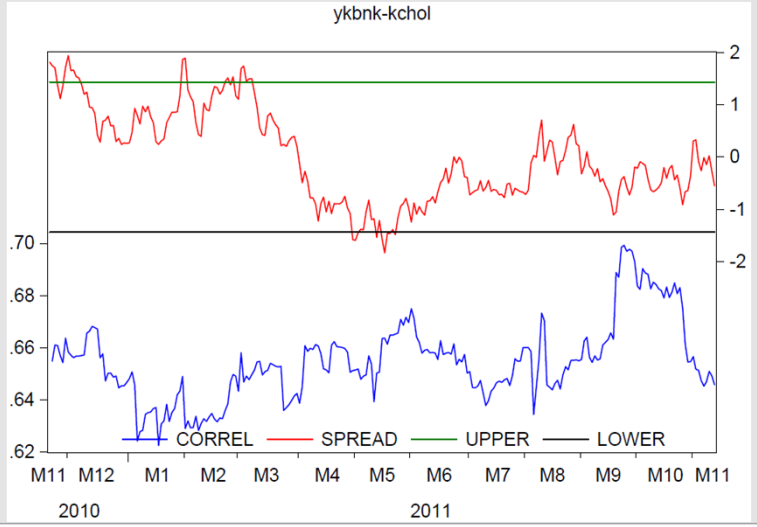
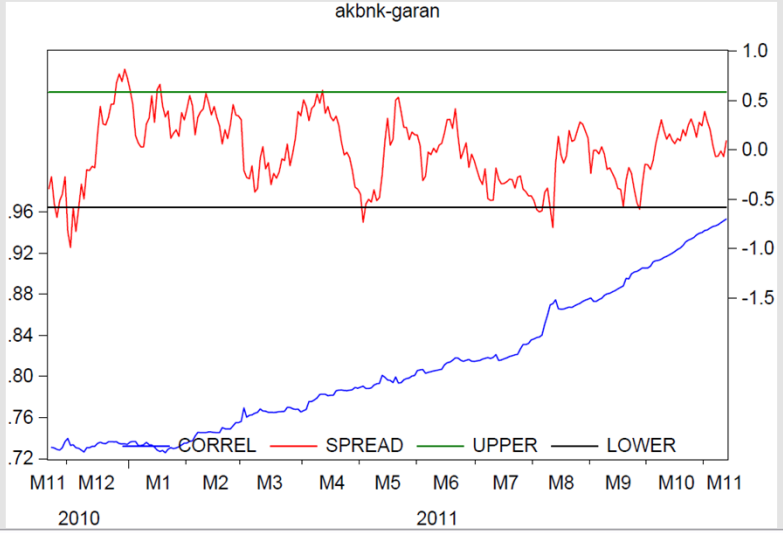
Sample Output (e.g. Dynamic Conditional Correl.)

	Final	Average	Quantile(q20)	Quantile(q80)
ykbnk-kchol	0.646	0.655	0.644	0.663
isgyo-sngyo	0.632	0.629	0.596	0.671
ttkom-tcell	0.149	0.266	0.155	0.402
akbnk-garan	0.953	0.808	0.737	0.875
akbnk-vakbn	0.796	0.759	0.714	0.797
akbnk-ykbnk	0.839	0.763	0.697	0.839
garan-vakbn	0.839	0.810	0.788	0.829
garan-ykbnk	0.823	0.826	0.782	0.873
vakbn-ykbnk	0.778	0.813	0.785	0.861





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A few remarks on summary results

- Although the analysis is designed to generate an output in any case, there is no guarantee that all the results will be meaningful.
- Some methods impose strict functional forms, which may lead to infeasible/incorrect results if the assumptions do not hold.
- Some methods, on the other hand, rely on more complex analyses and may therefore experience estimation problems.
- Charts and warning messages in the tables will be helpful in identifying the source of the problem.
- Much of the statistical output generated during the estimation process is intentionally left outside the scope of the summary report.
- The summary report, as an analytic tool, seeks to reflect the trade-offs among flexibility, customization, ease of use and capability.

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Essentially, all models are wrong, but some are useful.

George Edward Pelham Box

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